Linear Statistical Models

Week-4: Graded Assignment

Subjective Assignment: (Manual-grading) **Max. Marks: 25**

Note: *R is not required for this assignment.*

1. Consider a Linear Model:

$$y_{ij} = \mu + \mu_i + \epsilon_{ij} \quad ; \quad 1 \leq i \leq 2\ ,\ 1 \leq j \leq 2$$

Decide if the parameters μ and μ_i are identifiable from the model. If not then specify a condition for the same. [2 Marks]

2. Consider a model:

$$\begin{aligned} y_1 &= \beta_1 + \beta_2 + \beta_3 + \beta_4 + \epsilon_1 \quad ; \\ y_2 &= \beta_1 + \beta_3 - \beta_2 - \beta_4 + \epsilon_2 \quad ; \\ y_3 &= \beta_1 + \beta_2 - \beta_3 - \beta_4 + \epsilon_3 \quad ; \\ y_4 &= \beta_1 + \beta_4 - \beta_2 - \beta_3 + \epsilon_4 \end{aligned}$$

where, $\beta_i \in R$; $i = 1, 2, 3, 4$ and ϵ_i's $(i = 1, 2, 3, 4)$ are uncorrelated random variables with variance σ^2.

   (a) If we want to rewrite the model as $(\underset{\sim}{Y}, X\underset{\sim}{\beta}, I_{4\times 4})$, then find $\underset{\sim}{Y}, X, \underset{\sim}{\beta}$ and $\underset{\sim}{\epsilon}$. [2 Marks]

   (b) Write down the Normal equation for the above model. [3 Marks]

   (c) Using the normal equation obtained in part (c), find the least square estimates of $\beta_1, \beta_2, \beta_3$ and β_4. [4 Marks]

3. Consider a model:

$$\begin{aligned} y_1 &= 12\beta_1 + 6\beta_2 + \epsilon_1 \quad ; \\ y_2 &= 10\beta_1 - 2\beta_2 + \epsilon_2. \end{aligned}$$

where, ϵ_i's $(i = 1, 2)$ are uncorrelated random variables with variance 1.

   (a) Compute the expression for $\|\underset{\sim}{Y} - X\underset{\sim}{\beta}\|_2$. [2 Marks]

   (b) Let $\underset{\sim}{Y} = \begin{pmatrix} 48 \\ 12 \end{pmatrix}$, then for which of the following value of $\underset{\sim}{\beta}$ will the value of $\|\underset{\sim}{Y} - X\underset{\sim}{\beta}\|_2$ is minimum? [2 Marks]

(i). $\underset{\sim}{\beta} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(ii). $\underset{\sim}{\beta} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

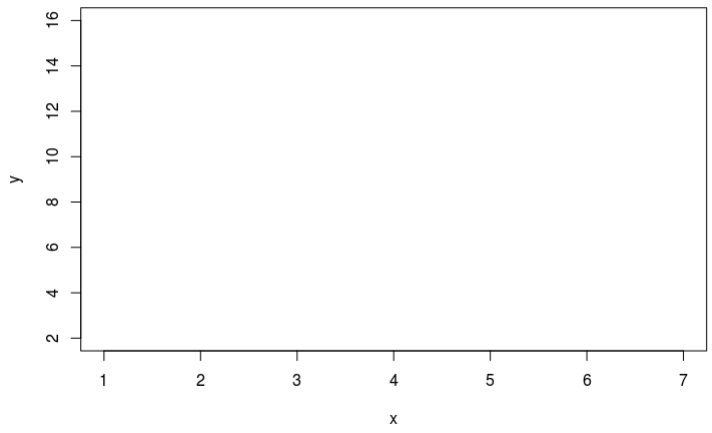
(iii). $\underset{\sim}{\beta} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(iv). $\underset{\sim}{\beta} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$

4. Consider the following data.

$\underset{\sim}{x}$	$\underset{\sim}{y}$
1	2
2	3
3	1
4	4
5	6
6	5
7	7

(a) Make a scatter plot of $(\underset{\sim}{x}, \underset{\sim}{y})$ in the graph below: [2 Marks]



(b) Suppose we assume the linear model $(\underset{\sim}{Y}, X\underset{\sim}{\beta}, I_{7\times 7})$ for the data obtained from the scatter plot. Find the least square estimate for $\underset{\sim}{\beta}$. [4 Marks]

5. Consider a 2×2 matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Find Column space (A), Row space (A), Null space (A) and Column space (A^T). [4 Marks]