Erdös Renyi Graph $G(n,p)$ is constructed in the following manner:

1. Consider n vertices labeled $\{1, 2, \ldots, n\}$.

2. Corresponding to each distinct pair $\{i, j\}$ we perform an independent Bernoulli (p) experiment and insert an edge between i and j with probability p. Note that all edges are *undirected* and hence there are total of $\binom{n}{2}$ possible edges, each occuring with probability p.

3. In this Homework you will simulate an Erdös Renyi Graph and find the M.L.E for the relevant p.

   1. Choosing x: Write a simple R-code to generate a number uniformly from $\{1, 2, 3, 4, 5\}$. Let x denote the chosen number. Record x in the box: [ ].

   2. Consider the experiment of rolling a die and (choose) specify an event from that experiment which occurs with probability $x/6$. Let it be called B. Write out the description of the event B in the box below:

      [ ].

   3. The set of vertices for the graph you are about to construct are $\{1, 2, \ldots, 10\}$. The graph has no self-edges (i.e Self-loops). What is the total number of possible edges ?

      Record answer in the box: [ ]

   4. Construct the *random* adjacency matrix A for the graph as follows. For each pair $1 \leq i < j \leq 10$: *roll the die*(using one at home or at http://www.randomservices.org/random/apps/Dice.html) and observe if the event B has occured.
      Designate

$$a_{ij} = \begin{cases} 1 & \text{if } B \text{ occured.} \\ 0 & \text{if } B \text{ did not occur} \end{cases}$$

Fill in the matrix entries accordingly:

$$
\begin{bmatrix}
0 & \square & \square & \square & \square & \square & \square & \square & \square & \square \\
 & 0 & \square & \square & \square & \square & \square & \square & \square & \square \\
 & & 0 & \square & \square & \square & \square & \square & \square & \square \\
 & & & 0 & \square & \square & \square & \square & \square & \square \\
 & & & & 0 & \square & \square & \square & \square & \square \\
 & & & & & 0 & \square & \square & \square & \square \\
 & & & & & & 0 & \square & \square & \square \\
 & & & & & & & 0 & \square & \square \\
 & & & & & & & & 0 & \square
\end{bmatrix}
$$

5. Using the `igraph` package draw the random graph , denote by $G(10, \frac{x}{6})$, corresponding to the above adjacency matrix (i.e draw an edge between i and j if $a_{ij} = 1$). *Send the image of your graph on zoom chat group between 8:15am and 8:30am on May 5th, 2022.*

6. From the graph $G(10, \frac{x}{6})$ or from A that you constructed in the worksheet:

   (a) fill in the following table from the data in worksheet:

x	# Edges

7. Let E denote the number of edges in a realisation of $G(10, \frac{x}{6})$. Find the likelihood $L(x; E)$ that E edges occur in the random Graph $G(10, \frac{x}{6})$.

8. Find x^* that maximizes $L(x; E)$ with respect to x. You may assume $x \in [1, 5]$.

9. Substitute your value of E from Question 1, into the expression for x^*. Is the resulting x^* close to your chosen x ?