

# Measure and Probability:(ISBN- 14 3980 126 6)

## Typos list

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**Errata List:** If you find any errors (typographical or otherwise) and they are not on the list below then please email us at athreya@isibang.ac.in or sunder@imsc.res.in. We will really appreciate it. Thanks.

**Page 2, line -12:** “:” should be replaced by “.”

**Page 6, line 11** “ $B, M, S$ ” should be “ $\mathcal{B}, \mathcal{M}, \mathcal{S}$ ”

**Page 7, line 10** Replace

$$\begin{aligned}\mathcal{M} &= \{A \in \mathcal{M}(\mathcal{A}) : \mathcal{M}_A = \mathcal{M}(\mathcal{A})\} \\ &= \{A \in \mathcal{M}(\mathcal{A}) : E \in \mathcal{M}(\mathcal{A}) \Rightarrow E \cup A \in \mathcal{M}(\mathcal{A})\}.\end{aligned}$$

by

$$\begin{aligned}\mathcal{M} &= \{A \in \mathcal{M}(\mathcal{A}) : \mathcal{M}_A = \mathcal{M}(\mathcal{A})\} \\ &= \{A \in \mathcal{M}(\mathcal{A}) : E \in \mathcal{M}(\mathcal{A}) \Rightarrow E \cup A \in \mathcal{M}(\mathcal{A})\}.\end{aligned}$$

**Page 43, Line -3:** “Let  $X, Y$  be two random variables ...” should be replaced by “Let  $X$  be a random variable ...”

**Page 45, Definition 3.2.1:** “ $P(\cap_{i=1}^n A_i) = \prod_{i=1}^n P(A_i)$ ” should be replaced by

$$P(\cap_{i=1}^n A_i^{\epsilon_i}) = \prod_{i=1}^n P(A_i^{\epsilon_i}),$$

where  $\epsilon_i \in \{0, 1\}$ ,  $A_i^0 := A_i$  and  $A_i^1 := A_i'$ .

**Page 70, paragraph 3:** In the proof of Kolmogorov’s Consistency Theorem (Th. 4.4.4), the third paragraph on page 70 should be changed to read as follows:

So, suppose  $\varepsilon > 0$ . Since, by assumption,  $P|\mathcal{B}_n$  is a probability measure, we can, by Proposition 4.4.2, find a compact set  $C_n \subseteq E_n$  such that  $P(A_n \setminus$

$B_n) \leq \varepsilon/2^{n+1}$ , where  $B_n = \pi_{n\downarrow}^{-1}(C_n)$ . Note that if  $\tilde{A}_n = \bigcap_{k=1}^n B_k$ , then  $\tilde{A}_n \in \mathcal{B}_{n\downarrow}$  and in fact  $\tilde{A}_n = \pi_{n\downarrow}^{-1}(K_n)$  where  $K_n = \{(x_1, \dots, x_n) \in C_n : (x_1, \dots, x_k) \in C_k \text{ for } 1 \leq k \leq n\}$  is a compact subset of  $\mathbb{R}^n$ . Now

$$\begin{aligned} P(A_n \setminus \tilde{A}_n) &= P(A_n \setminus \bigcap_{k=1}^n B_k) \\ &\leq \sum_{k=1}^n P(A_n \setminus B_k) \\ &\leq \sum_{k=1}^n P(A_k \setminus B_k) \\ &< \varepsilon/2 \end{aligned}$$

and hence,  $P(\tilde{A}_n) \geq \varepsilon/2$  for all  $n$ , and hence each  $K_n$  is a non-empty compact set.

**Page 75, (5.1.2):** should read as :

$$f_\sigma(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \phi_X(t) e^{-\frac{\sigma^2 t^2}{2}} dt$$

**Page 76, line 2:**

$$= \int_{\mathbb{R}} e^{-iax} \phi_X(y) e^{-\frac{\sigma^2 y^2}{2}} dy$$

should be changed to

$$= \frac{1}{2\pi} \int_{\mathbb{R}} e^{-iax} \phi_X(y) e^{-\frac{\sigma^2 y^2}{2}} dy$$

**Page 76, Lemma 5.1.7:**  $(\Omega, \mathcal{B})$  should be replaced by  $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$

**Page 76, line -1:** “real-valued” should be replaced by “non-negative”

**Page 83, line -6:** “ $\forall n \geq m$ ” should be changed to “ $\forall n \geq m_1$ ”

**Page 83, line -7:** “ $\forall n \geq m$ ” should be changed to “ $\forall n \geq m_1$ ”

**Page 85, line -9:** “ $1 \leq i \leq n$ ” should be changed to “ $1 \leq i \leq n - 1$ ”

**Page 93, line 11:** “ $A \in \mathcal{B}$ ” should be changed to “ $A \in \mathcal{F}$ ”

**Page 116, Line 1:** “ $f : \mathbb{N} \rightarrow \mathbb{R}$ ” should be changed to “ $f : S \rightarrow \mathbb{R}$ ”

**Page 140, (7.1.3):** should read as

$$E_1, E_2 \in \mathcal{B}, E_2 \in \mathcal{C}_F, E_1 \subset E_2 \Rightarrow E_1 \in \mathcal{C}_F.$$

**Page 140, line -6:** “.. in the notation of (3)..” should be changed to “..in the notation of (2)..”

**Page 150, line 4:** “ $X$ ” should be changed to “ $\Omega$ ”.

**Page 153, line -4:** should be changed to

$$\implies g1_E = 1_E \rho \text{ a.e.}$$

**Page 160, line 8:** should be changed to

$$|x - m_k| \leq \lambda(I) + \frac{1}{2}\lambda(I_k) \leq s_{k-1} + \frac{1}{2}\lambda(I_k) < 2\lambda(I_k) + \frac{1}{2}\lambda(I_k) = \frac{5}{2}\lambda(I_k).$$

**Page 160, line -7:** “ $\mathbb{R}$ ” should be changed to “ $[a, b]$ ” and “ $x \in \mathbb{R}$ ” should be changed to “ $x \in [a, b]$ ”

**Page 161, line 17:** “Proposition 4.4.1” should be changed to “Lemma 4.4.1”

**Page 161, line -3:** “such that  $J_j^y = (y - j, y + j) \subset I_i$ ” should be replaced by “such that  $J_j^y = (y - k_j, y + k_j) \subset I_i$ ”

**Page 161, line -1:**

$$f(y + j) - f(y) > qj,$$

should be changed to

$$f(y + j) - f(y) > qk_j,$$

**Page 162, line 1:** “for all  $j \leq k(y)$ .” should be changed to “for all  $k_j \leq k(y)$ .”

**Page 162, line 7:** “ $= p \sum_{k=1}^n h_k$ ” should be changed to “ $= 2p \sum_{k=1}^n h_k$ ”

**Page 163, line -1:** “ $N^a(x) := \sup_{\pi \in \mathcal{P}[a, x]} \sum_{i=1}^k -[(f(x_i) - f(x_{i-1}))^-]$ ” should be replaced by “ $N^a(x) := \sup_{\pi \in \mathcal{P}[a, x]} \sum_{i=1}^k [(f(x_i) - f(x_{i-1}))^-]$ ”

**Page 164, Proposition 7.5.7:** Here and for the rest of the section:  $\lambda$  denotes Lebesgue measure.