Statistics for Decision Making - II

Full Marks: 40 Time : 2 hr 30 minutes

Answer 1 and 2 and any two from the rest.

1. Consider the following data on the average yield in a chemical production process, based on two catalysts. Check whether the catalysts give similar mean yield.

Catalyst 1	91.5	94.18	92.18	95.39	91.79	89.07	94.72	89.21
Catalyst 2	89.19	90.95	90.46	93.21	97.19	97.04	91.07	92.75

$t_{0.05, 14}$= 1.761, $t_{0.025, 14}$= 2.145, $t_{0.05, 15}$= 1.753, $t_{0.025, 14}$= 2.131 (10)

2. a)Let X_i, i = 1(1)n be a random sample from a distribution with pdf

$$f(x) = \begin{cases} \frac{1}{\theta_2}\, e^{-(x-\theta_1)/\theta_2}, \theta_1 < x < \infty \\ 0, Otherwise \end{cases}$$

$-\infty < \theta_1 < \infty$ and $\theta_2 > 0$.

Find the sufficient statistic for the parameters. (5)

b) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the following pdf. Find the CRLB of an unbiased estimator of p. (5)

$$f(x) = \begin{cases} \frac{\alpha^p}{\Gamma(p)}\, e^{-\alpha x} x^{p-1}, x > 0, \alpha > 0, p > 0 \\ 0, Otherwise \end{cases}$$

3. a) Define Minimum variance unbiased estimator.

b) Let T_0 be an MVUE of an unknown parameter θ and T_1 be an unbiased estimator of θ, such that $\frac{V(T_0)}{V(T_1)} = e_\theta$. Then $\rho_\theta = \sqrt{e_\theta}$, where, $\rho_\theta = Corr(T_0, T_1)$. (3+7 = 10)

4. a) Define consistency of an estimator of a parameter or parametric function.

b) Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from $N(\mu, \sigma^2)$. Find the MME of σ^2. Check whether it is i) unbiased, ii) consistent for σ^2 . (2+ 3+ 3+2)

5. Define the following: i) level of significance, ii) critical region, iii) Type I and type II error (3 + 3 + 4)