Statistics for Decision Making - II

Full Marks: 50 Time : 3 hrs

Answer 1 to 4 and any 2 from the rest

1. Following data pertains to the lengths (in cm.) of some articles produced by a machine. Before making any further analysis, the quality engineer needs to ensure the randomness of the sample observations. Carry out the necessary test at 5% level of significance.

   68.2, 71.6, 69.3, 71.6, 70.4, 65, 63.6, 64.7, 65.3, 64.2, 67.6, 67.6, 66.8, 68.9, 66.8, 70.1, 61.2, 60.5, 63.1, 64.9, 67.8, 71.5, 70.6, 70.6, 68.9, 67.8, 64.5, 57.3 (5)

2. The marks obtained by 20 students of a college are given for tem 1 and term 2 exam for same batch. Do you think the scores are correlated?

College A				College B		
89	71	47	29	79	12	22
76	84	81	49	61	55	90
63	97	32	73	36	81	76
69	88	43	80	50	73	62
55	52	86	44	50	73	62

Assume the data to follow normal distribution (univariate or bivariate as the case may be).

At $\alpha = 0.05$, $t_{20} = 1.725$, $t_{19} = 1.729$, $t_{38} = 1.69$ (7)

3. The following data shows mistake per page in a book. Does the data follow Poisson distribution?

Mistakes per page	0	1	2	3
No. of pages	211	90	19	5

Pmf of Poisson distribution is $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,2, \ldots \lambda > 0$

Consider at $\alpha = 0.05$, $\chi^2_3 = 7.815$, $\chi^2_2 = 5.991$, $\chi^2_1 = 3.841$ (8)

4. Production of sugar(in quitals) in 3 machines are given below. Analyse the data and check whether the production varies over selection of machine or not.

   Machine I: 2.0, 2.2, 1.7
   Machine II: 1.8 2.2, 2.0
   Machine III: 3.0. 2.8. 3.2 (10)

   Consider $F_{0.05, 2, 6} = 5.14$, $t_{0.05, 6} = 1.943$

5. a) State and Prove Naman Pearson Lemma (7)

b) Define Power function of a test (3)

6. State and prove Cramer Rao lower bound stating all the regularity conditions. (10)

7. a) Let $X_1, X_2, \ldots X_n$ be a random sample from a Gamma distribution with pdf

$$f(x) = \frac{\alpha^p}{\Gamma(p)} e^{-\alpha x} x^{p-1} \; if \; x > 0$$

Find the joint sufficient statistics for α and p. (5)

b) Derive the expression for the interval estimate of the ratio of variances of two independent normal populations. (5)