

Indian Statistical Institute, Bangalore
M.S. (QMS) First Year
Second Semester – Statistics for Decision Making II

Mid Term Exam Duration: 2 Hrs Date: 24-02-2023 Max Marks: 50

This paper carries 60 Marks. Answer as many questions as you can.

1. A dietician likes to find out the effectiveness of the diet plan she provides for her patients. The plan is considered effective if the patient loses his/her weight by at least 2 kg in 90 days. Given the scenario, explain how you will conduct the study, the null and alternate hypotheses, the test statistics, etc. [10]

2. Let y_1, y_2, \dots, y_n be a random sample of n observations on a random variable Y with the probability density function

$$f(y) = \frac{1}{\theta^2} y e^{-y/\theta}, \quad 0 \leq y < \infty, \quad 0 < \theta < \infty$$

Find the maximum likelihood estimator for θ . [6]

3. In a random sample of 85 automobile engine crankshaft bearings, 10 have a surface finish that is rougher than the specifications allow. A modification is made in the surface finishing process and that, subsequently, a second random sample of 85 axle shafts is obtained. The number of defective shafts in this second sample is 8.

- a. Obtain an approximate 95% confidence interval on the difference in the proportion of defective bearings produced under the two processes.
- b. Based on confidence interval, can we say the modification is able to reduce number of defective? [6+2=8]

4. A die is thrown 60 times. The number of times it lands with 1, 2, 3, 4, 5 and 6 face up is 5, 8, 9, 8, 10 and 20, respectively. Based on the above can we say that the die is biased. [Use $\alpha = 0.05$] [8]

5. Engineers are making breakthroughs to create synthetic silk fibres that can improve everything from car bumpers to bullet-proof vests or to make artificial blood vessels. One research group reports the summary statistics for the toughness (MJ/m³) of processed fibres.

$$n = 18, \quad \bar{x} = 22.6, \quad s = 15.7$$

- a. Construct a 95% confidence interval for the mean toughness of these fibres. Assume that the population is normal.
- b. Interpret the meaning of confidence interval. [5+3=8]

6. The following are the average weekly losses of worker-hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation:

Plant:	1	2	3	4	5	6	7	8	9	10
Before:	45	73	46	124	33	57	83	34	26	17
After:	38	64	42	119	35	51	77	29	22	11

Use the 0.05 level of significance to test whether the safety program is effective. [8]

7. In the context of hypothesis test explain the following briefly
- a. Test statistics, confidence interval of parameter and p-value in rejecting H_0 .
 - b. Show that for a fixed sample size n , α (type-1 error) increases as β (type-2 error) decreases, and vice versa. [6+6=12]