Final Exam   **Date: May 04,2023**
Maximum marks: 50   **Duration: 3 hours**

Note: This paper carries 60 marks. Answer as many questions as you like.

1. Consider a simple Linear Regression Model
$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \qquad \text{for } i = 1 \text{ to } n$$
   Where y is the dependent variable, and x is the independent variable.
   a. Estimate the value of β_0 and β_1 by using the method of least square.
   b. Prove that the least square estimate of $\beta_1 (or \ \beta_0)$ is the unbiased estimator.

   [3+7=10]

2. Write down the assumption you made in developing a linear regression model. How do you check whether the assumptions are valid or not? What will be the remedial measures in case assumptions are not valid?

   [2+5+3=10]

3. Independent random samples of three different brands of magnetron tubes (the key components in microwave ovens) were subjected to stress testing, and the number of hours each operated without repair was recorded. Although these times do not represent typical life lengths, they indicate how well the tubes can withstand extreme stress. The data are given below. Experience has shown that the distributions of life lengths for manufactured products are often nonnormal. Use the appropriate test to determine whether evidence exists to conclude that the brands of magnetron tubes tend to differ in length of life under stress. Test using $\alpha = 0.05$.   [8]

Lengths of life for Magnetron Tubes

Brand		
A	**B**	**C**
36	49	71
48	33	31
5	60	89
67	2	58
53	55	42

4. A company operates four machines, three shifts each day. From production records, the following data on the number of breakdowns are collected:

Shift	Machines			
	A	B	C	D
1	41	20	12	16
2	31	11	9	14
3	15	17	16	10

From the above data, can we say that breakdowns are independent of the shifts?

   [8]

5. The number of defective units found each day by an in-circuit functional tester in a printed circuit board assembly process is shown below:

Number of defective per Day	Times Observed
Up to 10	6
10-15	11
15-20	16
20-25	28
25-30	22
30-35	19
35-40	11
40 and above	4

Is it reasonable to conclude that these data come from a normal distribution with a mean of 24.64 and a variance of 80.42? [10]

6. An experiment was conducted to determine whether either firing temperature or furnace position affects the baked density of a carbon anode. The data are as follows:

Position	Temperature		
	800	825	850
1	570	1063	565
	565	1080	510
	583	1043	590
2	528	988	526
	547	1026	538
	521	1004	532

[Hint: $\sum y_i^2 = 10031071$ and $\sum y_i = 12779$]

   a. State the hypotheses of interest.
   b. Test the above hypotheses using the analysis of variance with $\alpha = 0.05$. What are your conclusions? [2+6=8]

7. Two referees in a flower beauty competition rank the 10 types of flowers as follows:

Flower	Referee A	Referee B
1	1	3
2	6	8
3	5	4
4	10	9
5	3	6
6	2	2
7	4	1
8	9	10
9	7	5
10	8	7

Analyse that data to identify whether the referees are giving rank consistently or not.
[6]