Indian Statistical Institute, Bangalore
M.S. (QMS) First Year
Second Semester – Statistics for Decision Making II

Mid Term Exam Duration: 2 Hrs Date: Max Marks: 50

This paper carries 60 Marks. Answer as many questions as you can.

1. Let $y_1, y_2, \cdots, y_n$ be a random sample of *n* observations on a random variable *Y* mean μ and variance σ^2. Show that the sample variance s^2 is an unbiased estimator of the population variance σ^2 when the distribution of the sampled population is unknown.

   [10]

2. A PC manufacturer wants to evaluate the performance of its hard disk memory system. One measure of performance is the average time between failures of the disk drive. To estimate this value, a quality control engineer recorded the time between failures (follows Normal Distribution) for a random sample of 45 disk-drive failures. The following sample statistics were computed:

   $$\bar{y} = 1765 \text{ hrs and } s = 215 \text{ hrs}$$

   a. Estimate the true mean time between failures with a 95% confidence interval.
   b. If the hard disk memory system runs properly, the true mean time between failures will exceed 1,700 hours. What can you infer about the disk memory system based on the interval calculated at part (a)?

   [7+3=10]

3. Let $y_1, y_2, \cdots, y_n$ be a random sample of *n* observations on a random variable *Y* with the exponential density function

$$f(y) = \begin{cases} \dfrac{e^{-y/\beta}}{\beta} & \text{if } 0 \le y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

   Determine the maximum likelihood estimator of β. Is the estimator unbiased?

   [7+3=10]

4. A federal traffic safety engineer wants to ascertain the effect of wearing safety devices (shoulder harnesses, seat belts) on reaction times to peripheral stimuli. A study was designed as follows: A random sample of 15 student drivers was selected from students enrolled in a driver education program. Each student driver received two reaction-time scores, one for the restrained

condition and one for the unrestrained condition. The data (in hundredths of a second) are shown below. Find and interpret a 95% confidence interval for the difference between restrained and unrestrained drivers' mean reaction time scores.

[10]

Reaction time data for Exercise -4

Driver	Condition	
	Restrained	Unrestrained
1	36.7	36.1
2	37.5	35.8
3	39.3	38.4
4	44.0	41.7
5	38.4	38.3
6	43.1	42.6
7	36.2	33.6
8	40.6	40.7
9	34.9	32.5
10	31.7	30.7

5. Researchers feel that the chances of having girl children in a community are 50%. They have counted the number of girls in 100 families of 5 children to validate their theory. The distribution of the number of girls per family is given below. Test whether the (observed) distribution of the number of girls follows a binomial distribution (with p=0.5) or not.

Girls	0	1	2	3	4	5
Number of family	5	12	28	33	17	5

[10]

6. (a) Explain Type-I & Type-II error with an appropriate example.
(b) Suppose if we increase significance level (α) of a hypothesis test. Then how it will impact the power of test. Explain mathematically or intuitively.

[4+6=10]