

**M.S. (QMS) First Year  
Second Semester - Statistics for Decision Making-II**

*This paper has 60 Marks. Answer as many questions as you can, but the maximum score is limited to 50.*

1. Let  $y_1, y_2, \dots, y_n$  denote a random sample from a uniform distribution with probability density

$$f(y) = \begin{cases} 1 & \text{if } \theta \leq y \leq \theta + 1 \\ 0 & \text{elsewhere} \end{cases}$$

- a. Show that  $\bar{y}$  is a biased estimator of  $\theta$ , and compute the bias.
- b. Find  $V(\bar{y})$
- c. What function of  $\bar{y}$  is an unbiased estimator of  $\theta$ ?

**[4 + 4 + 2 = 10]**

2. (a) Show that for a fixed sample size  $n$ ,  $\alpha$  increases as  $\beta$  decreases, and vice versa.  
(b) The maximum likelihood estimator of the mean  $\mu$  of a normal distribution is the sample mean  $\bar{y}$ . Consider the model  $E(y) = \mu$ . Show that the least-squares estimator of  $\mu$  is also  $\bar{y}$ .

**[5 + 5 = 10]**

3. The viscosity of fibre is known to be approximately normally distributed, with standard deviation  $\sigma=0.25$  Cp. A random sample of 25 sample has a mean life of  $\bar{x} = 3$  Cp.
- a. Is there evidence to support the claim that mean viscosity exceeds 3.1 Cp? Use  $\alpha=0.05$ .
  - b. What is the p-value for the test in part (a)?
  - c. What is the  $\beta$ -error for the test in part (a) if the true mean viscosity is 3.05 Cp?

**[3 + 3 + 4 = 10]**

4. A field experiment was conducted to compare the effectiveness of different traps for catching beetles. Paraffin traps were baited either with or without linalool. Also, the colour of the trap was varied as green or yellow. Seven traps for each combination of bait and colour—a total of 28 traps—were set 1 meter from the ground in a random grid pattern. After 5 days, the total number of beetles captured by each trap was determined. The data (simulated) are displayed in the following table. The researchers are investigating the effect of bait type and colour on the mean number of beetles captured by the traps. Analyse the data for the researchers and draw conclusion.

	Yellow							Green						
With Linalool	17	22	13	15	14	18	19	0	2	0	0	0	1	1
Without Linalool	5	4	2	3	2	3	4	9	10	11	9	12	11	13

**[10]**

5. In simple linear regression model  $y = \beta_0 + \beta_1 + \varepsilon$ , the estimate the value of  $\beta_0$  and  $\beta_1$  through least-squares method. Prove that  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are the unbiased estimator of the intercept and slope respectively i.e.  $E(\hat{\beta}_0) = \beta_0$  and  $E(\hat{\beta}_1) = \beta_1$ .

[4 + 4 = 8]

6. An Electric Company has experimented with different means of evaluating the performance of solder joint inspectors. One approach involves comparing an individual inspector's classifications with those of the group of experts that comprise company's Work Standards Committee. In one experiment, 153 solder connections were evaluated by the committee and 111 were classified as acceptable. An inspector evaluated the same 153 connections and classified 124 as acceptable. Of the items rejected by the inspector, the committee agreed with 19. Conduct an appropriate test to see whether the inspectors evaluation method matching with committee's or not.

[6]

7. Mechanical science engineers studied the impact of infrasound on a person's blood pressure and heart rate. Ten university students were exposed to infrasound for 1 hour. The table gives the heart rate for each student, both before and after exposure. Is there any impact of infrasound on hear rate? Use  $\alpha = 0.05$ . [Hint: Normality assumption may not be correct]

[6]

Student	Before	After
1	60	73
2	60	73
3	60	79
4	79	86
5	78	66
6	74	68
7	69	80
8	76	84
9	77	86
10	64	76