Indian Statistical Institute, Bangalore
MS (QMS) First Year
Second Semester - Statistics for Decision Making II

Midterm Exam — Date: March 02, 2018
Maximum marks: 50 — Duration: 2 hours

Answer as many questions as you can, but the maximum score you can get is 50 only.

1. Suppose that X is a discrete random variable with the following probability mass function: where $0 \leq \theta \leq 1$ is a parameter.

X	0	1	2	3
P(X)	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

The following 10 independent observations were taken from such a distribution: (3,0,2,1,3,2,1,0,2,1). Find the maximum likelihood estimate of θ. [8]

2. The following are the weights, in grams, of 10 packages of grass seed distributed by a certain company:
46.4, 46.1, 45.8, 47.0, 46.1, 45.9, 45.8, 46.9, 45.2, and 46.0.
Find a 95% confidence interval for the variance of the weights of all such packages of grass seed distributed by this company, assuming a normal population. [7]

3. A manufacturer of a new pain relief tablet would like to demonstrate that its product works twice as fast as the competitor's product. Specifically, the manufacturer would like to test H_0: $(\mu_1 = 2\,\mu_2)$ Vs. H_1: $(\mu_1 > 2\,\mu_2)$ where μ_1 is the mean absorption time of the competitive product and μ_2 is the mean absorption time of the new product. Assuming that the variances σ_1^2 and σ_2^2 are known, develop a procedure for testing this hypothesis. [8]

4. Let **U1** and **U2** be independent random variables. Suppose that **U1** is **x2** with **v1** degrees of freedom while **U** = **U1 + U2** is chi-square with **v** degrees of freedom, where **v > v1.** Then prove that **U2** is chi-square random variable with **v − v1** degrees of freedom. [8]

5. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 Volts,and the manufacturer wishes to test $H0$: (μ =5 Volts) against $H1$: ($\mu \neq$ 5 Volts), using n = 8 units. [5+4=9]
(a) The acceptance region is 4.85≤ x-bar ≤ 5.15. Find the value of α .
(b) Find the power of the test for detecting a true mean output voltage of 5.1 Volts.

6. Explain the following with example: [4 x 5 = 20]
a) Unbiasedness
b) Efficiency
c) Method of Moments in Estimation
d) Type-I & Type-II error