Indian Statistical Institute, Bangalore

M.S (QMS) First Year

Second Semester – Statistics for Decision Making II

Final Exam Maximum Marks: 50 Duration: 3hrs Date: 27th Apr 2015

This paper has got questions for 60 marks. Answer as many questions as possible, but the maximum you can score is 50.

1.

a) It is known that a sample consisting of the values 12, 11.2, 13.5, 12.3, 13.8 and 11.9 comes from a population with the density function

$$f(x,\theta) = \begin{cases} \dfrac{\theta}{x^{\theta+1}}, & x > 1 \\ 0\ , & otherwise \end{cases}$$

Where $\theta > 0$. Find the maximum likelihood estimate of θ.

b) Explain what is meant by "95% confidence interval of shop floor rejection percentage is 5% $\pm$ 1.5%." [6+4 = 10]

2. In a shop study, a set of data was collected to determine whether or not the proportion of defectives produced was the same for workers on the day, evening and night shifts. The data collected are shown below:

Shift	Day	Evening	Night
Defectives	45	55	70
Non Defectives	905	890	870

Use α = 0.05 to determine if the proportion of defects is the same for all three shifts. [10]

3. In a simple linear regression model,

$y_i = \alpha + \beta x_i + \epsilon_i$, where $\epsilon_i \sim N\ (o, \sigma^2)\ i = 1, 2, \dots, n$

Show that

a) $E(\hat{\beta}) = \beta$

b) $V\ (\hat{\beta}) = \dfrac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})2}$ [4+6 = 10]

4. In one way ANOVA, for fixed effect model,

$$y_{ij} = \mu + \alpha_i + \in_{ij}, \begin{matrix} i = 1, 2, \dots, a \\ j = 1, 2, \dots, n \end{matrix}$$

Show that, $E(MSA) = \sigma^2 + \frac{n \sum_{i=1}^{n} \alpha_{i\ 2}}{a-1}$

(All terms having usual meaning) [10]

5. A study of the amount of rainfall and the quantity of air pollution removed produced the following data:

Daily Rainfall x (0.01cm)	4.3	4.5	5.9	5.6	6.1	5.2	3.8	2.1	7.5
Particulate Removed y $(\mu g/m^3)$	126	121	116	118	114	118	132	141	108

a) Find the equation of the regression line to predict the particulate removed from the amount of daily rainfall.
b) Estimate the amount of particulate removed when the daily rainfall is $x = 4.8\ units$.
c) Find the mean square error for estimating error variance.
d) Find the correlation coefficient between x and y. [4+1+3+2 = 10]

6. The following data represent the number of hours that two different types of scientific pocket calculators operate before a recharge is required.

Calculator A	5.5	5.6	6.3	4.6	5.3	5.0	6.2	5.8	5.1
Calculator B	3.8	4.8	4.3	4.2	4.0	4.9	4.5	5.2	4.5

Use Wilkoxon Rank Sum Test with $\alpha = 0.01$ to determine if Calculator A operates longer than Calculator B on a full battery charge. [10]