Indian Statistical Institute, Bangalore

M.S (QMS) First Year

Second Semester - Reliability, Maintainability and Safety II

Mid Term Exam

Time: 2.5 Hours Date: 19 February, 2025 Max Marks: 50

1. A manufacturing facility operates a high-temperature processing unit, which consists of four subsystems that must function to ensure uninterrupted production. The system architecture is as follows:

   Subsystem A (Cooling System) (Both pumps operate in parallel for redundancy.)

   • Pump 1 (R_{A1} = 0.92) • Pump 2 (R_{A2} = 0.92)

   Subsystem B (Monitoring System) (All sensors operate in series for accurate system monitoring.)

   • Sensor 1 (R_{B1} = 0.85) • Sensor 2 (R_{B2} = 0.85) • Sensor 3 (R_{B3} = 0.85)

   Subsystem C (Pressure Control System) (A single component controlling system pressure.)

   • Pressure Regulator (R_C = 0.88)

   Subsystem D (Power Supply System) (Both power units operate in parallel for backup power.)

   • Power Unit 1 (R_{D1} = 0.90) • Power Unit 2 (R_{D2} = 0.90)

   Overall System Configuration:
   (A in parallel with B) in series with (C in parallel with D)

   a) Evaluate the reliability of each subsystem separately, considering the series and parallel configurations.

   a) Calculate the overall system reliability.

   b) To improve reliability, an additional backup control sensor ($\boldsymbol{R_{B4}}$ = 0.85) is installed in Subsystem B. Determine the new system reliability and discuss the improvement's industrial significance.

   [3 + 5 + 7 = 15]

2. A reliability engineer at an industrial manufacturing plant is analysing the lifetime of a critical component used in high-temperature machinery. The organization wants to estimate the failure

characteristics of these components to optimize maintenance schedules and reduce downtime. The failure times (in hours) of a sample of components are assumed to follow a Weibull distribution with a known shape parameter $\alpha = 2$. However, due to operational constraints, some components were still functioning when the study was terminated, resulting in right-censored data. The observed failure and censoring times are:

2.3, 4.7, 5.1, 6.2+, 7.0, 3.8, 8.9+, 4.2, 5.6, 9.4, 6.1+, 7.3, 5.9, 8.1, 10.2+, 4.5, 3.6, 6.7+, 7.8, 8.5, 5.0, 9.0+, 6.4, 7.2, 10.5+, 3.9, 4.8, 6.0, 8.3, 9.7+ where a plus sign $(+)$ indicates a censored observation.

The failure times follow a Weibull distribution with probability density function:

$$f(t;\lambda) = \alpha\,\lambda\,t^{\alpha-1}e^{-\lambda\,t^{\alpha}}, \qquad t > 0.$$

where $\alpha = 2$ is known, and λ is unknown.

(a) Construct the likelihood function incorporating both the failure and censored data.
(b) Determine the Maximum Likelihood Estimate (MLE) of λ by solving the likelihood equation.
(c) If α were unknown, describe two alternative approaches to estimate both α and λ using the given data.

[5+10+5 = 20]

3. Explain the difference between HPP and NHPP models with the help of an example.

[5]

4. A system experiences failures according to a nonhomogeneous Poisson process (NHPP) with an intensity function given by: $\lambda(t) = 0.04 + 0.012t$, where t is in hours and $\lambda(t)$ represents the failure rate at time t.
(a) Determine the reliability $R(4,12)$.
(b) Determine the probability that no failures occur in the next 12 hours.

[5 + 5 = 10]