Indian Statistical Institute, Bangalore
M.S. (QMS) First Year

Second Semester - Reliability, Maintainability and Safety-II

Final Exam

Time: 3 hours Date: 25/04/2025 Maximum Marks: 50

1. Assume a repairable system has a Mean Time to Repair of 5 hours. The repair time follows an exponential distribution. Compute the probability that a failed system is repaired within 8 hours.

[5]

2. Explain the difference between design FMEA and process FMEA.

[5]

3. A mechanical component is subjected to a cyclic bending stress that is normally distributed with a mean of 45,000 *psi* and a standard deviation of 5,500 *psi*. A surface treatment process introduces a residual compressive stress that is normally distributed with a mean of 12,000 *psi* and a standard deviation of 2,000 *psi*. The material strength of the component is designed with a mean of 65,000 *psi*. However, due to uncertainties in the manufacturing process, the standard deviation of strength is unknown. To ensure that the component reliability does not fall below 98%, determine the maximum allowable standard deviation of strength.

[10]

4. Explain the difference between 'pro-rata warranty' and 'warranty with storage limitation'.

[5]

5. Explain the role of fault tree analysis in improving reliability of a system.

[5]

6. Consider a constant-stress accelerated life testing experiment with three stress levels, denoted as s_i $(i = 1,2,3)$. At each stress level s_i, there are n_i units and r_i failures are observed. Assume that the failure times follow exponential distribution with a scale parameter λ_i.

(a) Derive the maximum likelihood estimators for the parameters of the lifetime distribution under this setup.

(b) Now, assume the scale parameter follows a log-linear function of stress, with a normal operating condition stress level s_0. Construct the likelihood function and obtain the corresponding MLEs.

(c) Using the estimated parameters from part (b) above, how can we derive the estimate of reliability function $\widehat{R_0(t)}$ at the normal operating condition s_0?

[5+3+2=10]

7. Consider a step-stress accelerated life test where n identical units are subjected to an initial stress level s_1. The experiment continues until a specified number of failures r is observed. Let τ be the fixed (known) time at which the stress level is increased from s_1 to s_2. A total of n_1 failures occur before τ, where $1 \leq n_1 \leq r - 1$ and the test continues until a total of r failures is observed. Let $t_{1:n} < t_{2:n} < \cdots < t_{r:n}$ denote the ordered failure times. Assume the cumulative exposure model, and the lifetime distributions at stress levels s_1 and s_2 are exponential with means θ_1 and θ_2, respectively.

(a) Construct the likelihood function for this step-stress ALT.

(b) Derive the maximum likelihood estimates of θ_1 and θ_2.

[6+4= 10]