Indian Statistical Institute, Bangalore

M.S (QMS) First Year

First Semester - Reliability, Maintainability and Safety I

Mid Term Exam Time: 2 Hours Date: 10 September, 2025 Max Marks: 50

Answer as many questions as you can. The maximum you can score is 50. No marks will be awarded if variables are not clearly defined or if justifications are not provided.

1. During the July 2024 dengue outbreak in Bengaluru, patients arrive randomly at a rescue camp. On average, one patient arrives every minute. This setup can be studied using two familiar ideas:
   - The Exponential distribution describes the time between two successive arrivals.
   - The Poisson distribution describes the number of arrivals in a fixed time interval.

   Using these facts, answer the following:
   a. What is the probability that the 10th patient arrives two or more minutes after the arrival of the 9th patient?
   b. What is the probability that the 10th patient arrives after 00:20 a.m. (20 minutes after midnight)? Equivalently, by 20 minutes there are fewer than 10 arrivals.
   c. During the early morning hours:
      - 2 patients arrive between 01:01 a.m. and 01:03 a.m.
      - 1 patient arrives between 01:03 a.m. and 01:04 a.m.
      - 2 patients arrive between 01:04 a.m. and 01:05 a.m.

   What is the probability that these arrivals happen exactly as described? (3+5+3)

2. Team X and Team Y play a series of independent games until one of them wins 4 games. The probability that Team X wins any single game is 60%.
   a. Find the probability that the series ends in at most 6 games.
   b. Given that the series ends in exactly 6 games, what is the probability that Team X won the series? (3+3)

3. Three friends — A, B, and C — go to a cinema hall where seats are pre-assigned.

   - A loses the ticket and chooses one of the three seats at random.
   - If B's own seat is empty, B sits there; otherwise, B chooses one of the remaining seats at random.
   - Finally, C takes the last remaining seat.

   Find the probability that C sits in the assigned seat. (6)

4. Let X be a discrete random variable taking values in {1,2, 3, …}. Suppose X satisfies the memoryless property, that is, for all non-negative integers m and n:

$$P(X > m + n \mid X > m) = P(X > n)$$

Show that any discrete random variable satisfying this property must follow a Geometric distribution. (7)

5. Consider a system with four components, C_1, C_2, C_3, C_4, whose states are represented by $X_i = 1$ if the component C_i is functioning and $X_i = 0$ if it has failed. The system structure function is given by:

$$\phi(X_1, X_2, X_3, X_4) = X_1 X_2 + X_3 X_4 - X_1 X_2 X_3 X_4$$

   a. Draw the reliability block diagram of the system.
   b. Identify the minimal path sets and minimal cut sets for this system.
   c. If all components are independent and each has reliability $p_i = \frac{90+2i}{100}$, compute the overall system reliability R_s. (3+4+3)

6. The structure functions of two systems are given in the following table:

X_1	X_2	X_3	X_4	ϕ_1	ϕ_2		X_1	X_2	X_3	X_4	ϕ_1	ϕ_2
0	0	0	0	0	0		1	0	0	0	0	1
0	0	0	1	0	0		1	0	0	1	0	0
0	0	1	0	0	1		1	0	1	0	0	0
0	0	1	1	0	0		1	0	1	1	1	1
0	1	0	0	0	1		1	1	0	0	0	0
0	1	0	1	0	0		1	1	0	1	1	1
0	1	1	0	0	0		1	1	1	0	1	0
0	1	1	1	0	0		1	1	1	1	1	1

   a. Determine which system is coherent and which is non-coherent. Justify your answer.
   b. Draw the reliability block diagram of the coherent system.
   c. Identify the minimal path sets (MPS) and minimal cut sets (MCS) of the coherent system.
   d. If the component reliabilities are $R_1 = 0.99$, $R_2 = 0.85$, $R_3 = 0.90$, $R_4 = 0.75$, compute the system reliability. (5+3+5+2)