Indian Statistical Institute, Bangalore

M.S (QMS) First Year

First Semester - Reliability, Maintainability and Safety I

End Term Exam Time: 2 Hours Date: 12 November, 2025 Max Marks: 50

Answer as many questions as you can. The maximum you can score is 50.

1. A system consists of two components that are exposed to three independent shock processes. Shock 1 affects only Component 1, Shock 2 affects only Component 2, and Shock 3 affects both components simultaneously. This type of shock model leads to Marshall–Olkin bivariate distributions.

   Each shock arrival time follows a Weibull distribution with parameters $(\alpha\ ,\ \lambda_i\ )$ for $i = 1,2,3$, and all shocks occur independently. Each component fails immediately upon receiving a shock that affects it.

   a. Derive the joint reliability function of the components.
   b. Obtain the corresponding joint probability density function (PDF) of the components.
   c. Determine the distribution of the system lifetime when the system fails upon the failure of the first component (series system).

(5+5+5)

2. In a high-volume production process, the probability that an item is defective varies each day due to changes in temperature and humidity. Let this daily defect probability be a random variable P, follows Normal distribution with mean 0.1 and standard deviation 0.02, and assume that the number of defective items in a random sample of $n = 20$ items on a given day follows a Binomial distribution conditional on P: $X \mid P = p \sim Binomial(20, p)$.

   a. Using the law of total expectation $(E[X] = E_P\big[E_X[X \mid P]\big])$, find the expected number of defective items, $E[X]$.
   b. Using the law of total variance $(Var(X) = E_P[Var_X(X \mid P)] + Var_P(E_X[X \mid P]))$, find the variance of X, i.e., $Var(X)$.

(5+10)

3. If all the components of a series system have increasing (or decreasing) hazard rates and the components are independent, prove that the system lifetime also possesses an increasing (or decreasing) hazard rate property.

(10)

4. The B_p life of a component or system is the time by which p % of the population is expected to fail. The lifetime (in thousands of hours) of a certain type of industrial pump is approximately lognormally distributed with parameters $\mu = 6.0$ and $\sigma = 0.5$. Find the B_{10} life of the pump.

(5)

5. A shipping company is monitoring the number of trucks arriving at two nearby warehouses, Warehouse X and Warehouse Y, in a 5-minute interval. Let X and Y represent the number of trucks arriving at Warehouse X and Warehouse Y, respectively. The joint PMF is given by:

$$f(x, y) = c(1 + x + y + xy); x, y = 0,1,2.$$

   a. Find the mean and variance of $X + Y$, the total number of trucks arriving at both warehouses.
   b. Determine the correlation between X and Y.

(8+7)