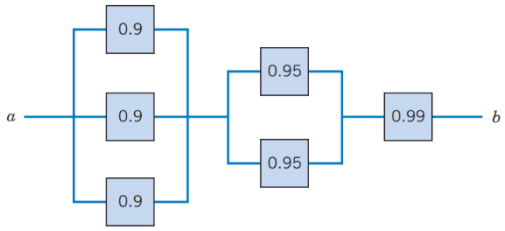
Indian Statistical Institute, Bangalore

M.S (QMS) First Year

First Semester - Reliability, Maintainability and Safety I

Mid Term Exam Time: 2.5 Hours Date: 09 September, 2024 Max Marks: 50

1. The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates? [6]



2. Natural red hair consists of two genes. People with red hair have two dominant genes, two regressive genes, or one dominant and one regressive gene. A group of 1000 people was categorized as follows:

	Gene 1		
	Dominant	Regressive	Other
Gene 2			
Dominant	5	25	30
Regressive	7	63	35
Other	20	15	800

Let A denote the event that a person has a dominant red hair gene and let B denote the event that a person has a regressive red hair gene. If a person is selected at random from this group, compute the following. (a) $P(A)$ (b) $P(A' \cap B)$ (c) Probability that the selected person has red hair. [3+3+3]

3. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline. (a) What is the probability that for exactly three calls the lines are occupied? (b) What is the probability that for at least one call the lines are not occupied? (c) What is the expected number of calls in which the lines are all occupied? [3+3+3]

4. Magnetic tape is slit into half-inch width that are wound into cartridges. A slitter assembly contains 48 blades. Five blades are selected at random and evaluated each day for sharpness. If any dull blade is found, the assembly is replaced with a newly sharpened set of blades. (a) If 10 of the blades in an assembly are dull, what is the probability that the assembly is replaced the first day it is evaluated? (b) If 10 of the

blades in an assembly are dull, what is the probability that the assembly is not replaced until the third day of evaluation. Assume the daily decisions are independent. [3+3]

5. When a computer disk manufacturer tests a disk, it writes to the disk and then tests it using a certifier. The certifier counts the number of missing pulses or errors. The number of errors on a test area on a disk has a Poisson distribution with mean= 0.2. (a) What is the expected number of errors per test area? (b) What percentage of test areas have two or fewer errors? [3+3]

6. The probability density function for the diameter of a drilled hole in millimeters is $10\ e^{-10(x-5)}$ for $x > 5$mm. Although the target diameter is 5 millimeters, vibrations, tool wear, and other nuisances produce diameters larger than 5 millimeters. (a) Determine the mean and variance of the diameter of the holes. (b) Determine the probability that a diameter exceeds 5.1 millimeters. [3+3]

7. A study in the Archives of Environmental & Occupational Health considered polycyclic aromatic hydrocarbons and immune system function in beef cattle. The mean monthly exposure to PM1.0 (particulate matter that is $< 1\ \mu$m in diameter) was approximately 7.1 $\mu g/m^3$ with standard deviation 1.5. Assume the monthly exposure is normally distributed. (a) What is the probability of a monthly exposure greater than 9 $\mu g/m^3$? (b) What is the probability of a monthly exposure between 3 and 8 $\mu g/m^3$? (c) What is the monthly exposure level that is exceeded with probability 0.05? (d) What value of mean monthly exposure is needed so that the probability of a monthly exposure greater than 9 $\mu g/m^3$ is 0.01? [2+2+2+2]

8. The maximum time to complete a task in a project is 2.5 days. Suppose that the completion time as a proportion of this maximum is a beta random variable with 2 and 3. What is the probability that the task requires more than two days to complete? [5]