Indian Statistical Institute, Bangalore

M.S (QMS) First Year

First Semester - Reliability, Maintainability and Safety I

Final Exam Time: 2.5 Hours Date: 06 November, 2024 Max Marks: 50

1. A hospital relies on a life-support system composed of four essential components. Each component can be either functioning (1) or failed (0), and the overall status of the life-support system depends on the combined state of these components. The table summarizes the operational status of the system based on different combinations of component states. Based on this table:

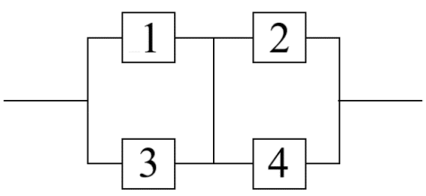
x_1	x_2	x_3	x_4	$\phi(\boldsymbol{x})$	x_1	x_2	x_3	x_4	$\phi(\boldsymbol{x})$
0	0	0	0	0	0	1	1	0	1
1	0	0	0	0	0	1	0	1	1
0	1	0	0	0	0	0	1	1	1
0	0	1	0	0	1	1	1	0	1
0	0	0	1	1	1	1	0	1	1
1	1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	1	1	1
1	0	0	1	1	1	1	1	1	1

   A) Draw a reliability block diagram to visually represent how each component impacts the system's operation.
   B) Determine the minimum critical component groups that would cause system failure if they all fail and the combinations of minimal functioning components that keep the system operational.
   C) Derive the structure function based on the table, showing the system's status depending on the components' states.
   D) If each component has hazard rate $\alpha\lambda_i^{\alpha}t^{\alpha-1}$ and all components operate independently, compute the reliability of the entire system. (2+4+3+6)

2. A manufacturing plant relies on a machine with 10 essential components, each with a hazard rate λ_i. The plant manager wants to know the machine's overall hazard rate when each component has a constant hazard rate.
   If each component's hazard rate instead follows $0.6\lambda_i t^{-0.4}$, how does this affect the overall hazard rate of the machine? Compare the system's hazard rate under both conditions. (3+3)

3. A cloud service provider, Rackspace, operates a system comprising 66 servers to host critical applications. The system is designed to function only if at least 22 servers are operational at any given time. To ensure high availability, the provider aims for the system reliability to be at least 95%.
   To address this reliability requirement, the provider hires you as an expert to derive the necessary reliability equation for the system. Formulate the inequality that must be satisfied to guarantee that the overall system reliability meets or exceeds 95%. (3)

4. A data center is designed with redundant network paths to ensure continuous connectivity for critical applications. The network has four primary routers, arranged in a bridge configuration as shown in the diagram. The routers are configured as follows:



Each router has an individual reliability, r, which indicates the probability that it will be functional at any given time. Derive an equation for the overall network reliability based on the given bridge configuration of routers. (6)

5. A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let X and Y represent the weights of the creams and the toffees, respectively, and suppose that the joint density function of these variables is

$$f(x,y) = 24xy\,; x + y \leq 1.$$

(a) Find the probability that in a given box the cordials account for more than half of the weight.
(b) Find the marginal density for the weight of the creams.
(c) Find the probability that the weight of the toffees in a box is less than 1/8 of a kilogram if it is known that creams constitute 3/4 of the weight. (3+3+3)

6. A logistics company is analyzing the number of delivery trucks that arrive at two different distribution centers (Center A and Center B) during a certain 5-minute period. These centers are located near each other, so the company's logistics team wants to consider them jointly to optimize resource allocation. Let X and Y represent the number of delivery trucks that arrive at Center A and Center B, respectively, during this period. The joint distribution of X and Y is given by:

$$f(x,y) = \frac{9}{16}\frac{1}{4^{x+y}}\,; x, y = 0,1,2, \ldots$$

(a) Find the mean and variance of X+Y, the total number of trucks arriving at both centers.
(b) Determine the correlation between X and Y. (3+3)

7. A telecommunications company is analyzing signal strength fluctuations at two nearby communication towers, Tower A and Tower B. Due to environmental factors and interference, the signal strength at each tower fluctuates randomly, and the fluctuations at the two towers are correlated.
Let X represent the signal fluctuation (in decibels) at Tower A, and Y represent the signal fluctuation at Tower B. The joint probability density function (PDF) of the signal fluctuations is given by:

$$f(x,y) = ce^{-8x^2-6xy-18y^2}; x, y \in \mathbb{R}.$$

Find the value of the constant c. Also, find the means, variances, and the correlation coefficient of X and Y. (3+3)

8. A manufacturing company produces two types of electronic components, Component A and Component B, which are often used together in various devices. The quality control team has established a joint density function that models the relationship between the dimensions of these components, specifically their lengths and widths. The joint density function is given by:

$$f(x,y) = \frac{16y}{x^3}\,; x > 2\,, y < 1,$$

where X represents the length (in cm) of Component A, Y represents the width of Component B. Given the joint density function, calculate the correlation coefficient to assess the strength and direction of the linear relationship between the lengths of Component A and the widths of Component B. (4)