

Indian Statistical Institute, Bangalore  
MS (QMS) First Year  
First Semester - Reliability, Maintainability and Safety I

Final Exam  
Maximum marks: 100

Date: March 22, 2021  
Duration: 3 hours

*Answer as many questions as you can.*

---

**Question (1):** Prove that  $P(\varnothing) = 0$  for any sample space “S” using axioms of Probability  
(6)

**Question (2):** Assume that  $P(A)$  not equal to zero and  $P(B)$  not equal to zero. Then A and B independent implies that they are not mutually exclusive and A & B mutually exclusive implies that they are not independent  
(3+3=6)

**Question (3):** If X is a random variable with distribution function  $F_x(t)$ . Define  $Y=a+bX$  where  $b>0$ . Then  $F_y(t) = F_x[(t-a)/b]$  and  $F_y(a+bt) = F_x(t)$  for all ‘ t’  
(7)

**Question (4):** The probability of successful optical alignment in the assembly of an optical data storage product is 0.8. Assume the trials are independent and Y be the no. of trials required for successful alignment

- (a) What is distribution of Y? Write down the pmf of Y
- (b) What is the probability that it takes less than 5 trials for a successful alignment?
- (c) Calculate the probability that the first successful trial requires an even number

(3+3+6=12)

**Question (5):** Show that in order to improve the reliability of a component or a system following a warranty period ‘ $T_0$ ’, hazard rate must be decreasing  
(8)

**Question (6):** Breakdowns in equipment at a large industrial plant have been observed to be approximately a Poisson Process with parameter  $\lambda=1/2$  per hour (i.e 1 every 2 hrs). If we arrive at this plant at 9 am on a Monday morning and “T” be the time (from our arrival) until the first breakdown, then

- (a) Write down the density function of “T”
- (b) What is the probability that it is at least 1 hour until the first breakdown?
- (c) What is the probability that it is no more than 4 hrs until the first breakdown?
- (d) What is the average time to the first breakdown?
- (e) Calculate the probability that the time to the next breakdown is greater than the average

(3+3+3+2+3=14)

**Question (7):** For the hazard function of the random variable time "t",  $h(t) = c.t$  where 'c' is a positive constant, find out the pdf  $f(t)$  and the reliability function  $R(t)$ . Which portion of the 'bath tub curve' is represented by this hazard function? What is the expected value of the r.v 't' ?

(4+2+2+7=15)

**Question (8):** In case of Three Parameter Weibull distribution

- (a) Show the relationship between characteristic life and median Life
- (b) Explain why the scale parameter is the life by which 63.2% of the population will have failed

(5+5=10)

**Question (9):** Consider a series system of 's' components. Each component follows Weibull distribution with the same shape parameter ' $\beta$ ' but differing scale parameter ' $\theta$ ' values

- (a) Derive an expression of the Hazard Function of the Series System
- (b) If all the 's' components have the same ' $\theta$ ', then what is the simplified hazard function

(6+4=10)

**Question (10):** Show that for a parallel system having Two Components and both the components have the same and constant failure rate, the MTBF of the system is equal to 1.5 times MTBF of a single component

(8)

**Question(11):** Consider a Two Unit Standby system with Imperfect Switching i.e the switch simply fails to operate when called upon. It is assumed that (a) both the units have constant failure rate  $\lambda$  (b) the switch is a complex piece of equipment and has a constant failure rate of  $\lambda_s$ , then find out that the reliability of the system.

(12)