

Indian Statistical Institute

M.S.(QMBA) First Year

First Semester – Probability

Mid Term Exam Date: 08/09/2026 Time: 2 hours Maximum Marks: 50

1. A random variable X has the following probability distribution:

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.3	0.25	0.15

- a) Verify that the above is a valid probability mass function.
b) Determine $V(X)$.

[4 + 6 = 10]

2. The diameter of a precision component is normally distributed with mean 50 mm and standard deviation 0.25 mm. The specification limits are 49.6 mm and 50.4 mm.
a) Determine the proportion of components that will be rejected.
b) Assuming that the process remains centered at a mean of 50 mm with standard deviation of 0.25 mm, determine the symmetric specification limits about the mean required to ensure that at least 99% of the components meet the specifications.

[4 + 6 = 10]

3. The number of surface defects on a sheet of material follows a Poisson distribution with parameter λ defects per sheet. It is known that the probability of finding no defects on a randomly selected sheet is equal to the probability of finding exactly one defect on the sheet.
a) Determine the value of λ .
b) Three sheets are selected independently. Determine the probability that at least one of the three sheets contains two or more defects.

[3 + 4 = 7]

4. The mean and variance of a Binomial distribution are 4 and $4/3$, respectively. Find $P(X \geq 1)$

[3]

5. A manufacturing plant has three production lines A, B, and C, which produce 30%, 45%, and 25% of the total production, respectively. The percentages of defective items produced by the three lines are 2%, 3%, and 5%, respectively.
- a) If an item is selected at random, what is the probability that it is defective?
- b) If the selected item is found to be defective, what is the probability that it was produced by production line B?

[3 + 3 = 6]

6. Find the expected number of throws of a fair die until the number 6 is obtained.
- [4]
7. The lifetime, in hours, of an electronic component follows an exponential distribution with a mean life of 500 hours. A component has already operated successfully for 400 hours, determine the probability that it will operate for at least another 300 hours.
- [5]
8. If X and Y are independent Poisson variates such that $P(X = 1) = P(X = 2)$ and $P(Y = 2) = P(Y = 3)$. Find the variance of $X - 2Y$.

[5]