Indian Statistical Institute, Bangalore
MS (QMS) First Year
Second Semester - Operations Research II

Final Exam — Date: May 05, 2018
Maximum marks: 100 — Duration: 3 hours

1. A small maintenance project consists of the following ten jobs (activities) whose precedence relationships are identified by their node numbers: [18]

Job name	Alternate (initial node, final node)	Estimated duration (days)
A	(1, 2)	2
B	(2, 3)	3
C	(2, 4)	5
D	(3, 5)	4
E	(3, 6)	1
F	(4, 6)	6
G	(4, 7)	2
H	(5, 8)	8
I	(6, 8)	7
J	(7, 8)	4

a) Draw an arrow diagram (network diagram) representing the project.
b) Calculate Earliest & Latest Occurrence time of each event (node)
c) How much Floats (TF) Job (3, 5) have? Job (4, 6)? Job (7, 8)?
d) What is the Critical Path?
e) If job (2, 3) were to take six days instead of three, how would the project finish date be affected?
f) Do any job have Free Float (FF)? If so, which one & how much?

2. [8 + 10 = 18]

(a) In the deterministic EOQ Inventory Model with price breaks, given that the inventory items may be purchased at discounted rate if the order quantity y exceeds a given limit q, mathematically develop the criteria for the optimal order quantity y*. You may use the following standard terminology: D == demand rate (units per unit time), K = set up cost per order, h = holding cost per unit inventory per unit time. Assume that no shortage is allowed. You may make any other assumption that may be required.

(b) Consider the inventory situation in which the stock is replenished uniformly (rather than instantaneously) at the rate a. Consumption occur at the constant rate D. Because consumption also occurs during the replenishment period, it is necessary that $a > D$. The setup cost is K per order, and the holding cost is h per unit per unit time. If y is the order size and no shortage is allowed, show that:

(i) The maximum inventory level is $y\,(1 - D/a)$
(ii) The total Cost per unit time is given y is $TCU(y) = KD/y + h/2\,(1 - D/a)$
(iii) The economic order quantity is $y^* = [2KD/h\,(1 - D/a)]^{1/2}$

3. [8 + 8 = 16]

(a) For the (M/M/c):(GD/∞/ ∞) Queueing Model with c parallel server, λ arrival rate & μ service rate, derive the mathematical formula for

- (i) p_n = Probability of n customer in the system
- (ii) p_0 = Probability of no customer in the system
- (iii) Expected queue Length L_q.

(b) A small town is being serviced by two cab companies. Each of the two companies owns two cabs and are known to share the market almost equally. This is evident by the fact that calls arrive at each company's despatching office at the rate of 10 per hour. The average time per ride is 11.5 minutes.

The two companies have been bought by an investor and will be consolidated into a single dispatching office.

Analyze the situation to find out whether the expected waiting time, W_q shows some improvement as a result of this consolidation. Also calculate the percentage of time of all cabs in the consolidated operations are "on call"

4. [6 + 10 = 16]

(a) Suppose that you have 7 full wine bottles, 7 half-full, and 7 empty. You would like to divide the 21 bottles among the three individuals so that each will receive exactly 7. Additionally, each individual must receive the same quantity of wine. Express the problem as Integer Linear Programing Problem. Use a dummy objective function with zero objective coefficients.

(b). Solve the following ILP Problem. You may use Branch & Bound method or any other method.

Maximize: $31x_1 + 9x_2$

Subject to: $-2x_1 + 5x_2 \le 20$
$21x_1 + 6x_2 \le 85$
x_1, x_2 are nonnegative Integers.

5. [7 + 9 = 16]

(a) Describe the methodology of generating, through simulation, a random observation X that follows an Exponential Distribution with parameter λ, taking as input, a random observation R from the Uniform distribution: $R \sim U(0, 1)$.
Now, randomly select 2 observations from the uniform (0, 1) random number table provided & use them to generate the exponential data for random variable $X \sim \exp(\lambda = 6)$.

(b) A bakery keeps stock of a popular brand of cake. Previous experience shows the daily demand pattern for the item with associated probabilities, as given below:

Daily Demand (number):	0	10	20	30	40	50
Probability :	0.01	0.20	0.15	0.50	0.12	0.02

Using simulation technique, simulate the demand for the next 10 days & also estimate the average daily demand on the basis of this 10 days simulated data. Use Random Number table provided to generate necessary input data.

6. An electronic device consist of three components. The three components are in series so that the failure of one component causes the failure of the device. The reliability,(i.e. probability of no failure) of the device can be improved by installing one or two standby units in each component. The table listed below charts the reliability, *r,* and the cost *c*. The total capital available for the construction of the device is $10,000. How should the device be constructed? Objective is to maximize the reliability $r_1r_2r_3$ of the device. [14]

Number of parallel units	Component 1 r_1	c_1 ($)	Component 1 r_2	c_1 ($)	Component 1 r_3	c_1 ($)
1	.6	1000	.7	3000	.5	2000
2	.8	2000	.8	5000	.7	4000
3	.9	3000	.9	6000	.9	5000

7. Two products are manufactured by passing sequentially through two different machines. The time available for the two products on each machine is limited to 8 hours daily but may be exceeded by up to 4 hours on an overtime basis. Each overtime hour will cost additional $5. The production rates for the two products together with their profits per unit are summarized below. It is required to determine the production level for each product that will maximize the net profit. Formulate this problem as a Goal Programming Model. [10]

Machine	Production Rate Product 1	Product 2
1	5	6
2	4	8
Profit per unit	$6	$4

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