

**Indian Statistical Institute, Bangalore**

MS (QMS) First Year

Second Semester - Operations Research II

Final Exam      Duration : 3 hours      Max Marks 100      Date : April 24, 2017

Remark: Answer Q4 and Q6 and any four from remaining questions. Maximum you can score is 100 only.

1. Solve the following integer programming problem

Maximize  $X_1 + X_2$

Subject to

$$7X_1 - 5X_2 \leq 7$$

$$-12X_1 + 15X_2 \leq 7$$

$X_1, X_2 \geq 0$  and integer.

(or)

You may Use the Branch and Bound technique to obtain optimal solution to the above problem. [15]

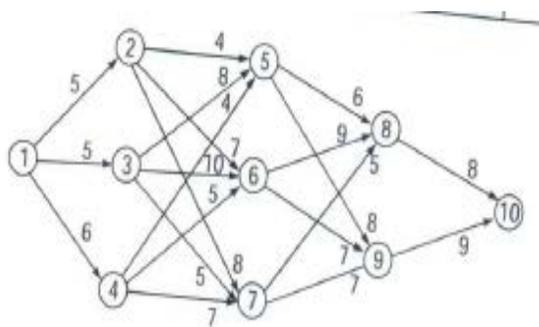
2. Let  $p$  among  $m$  points be chosen as locations where plants can be located. Let there be  $n$  demand points each having demand  $b_j$ . Plant  $i$  can produce  $a_i$  quantity of the product. There is a transportation cost of  $d_{ij}$  between plant  $i$  and destination  $j$ . There is a fixed cost  $f_i$  of setting a plant in location  $i$ . Let  $Y_i = 1$  if point  $i$  is chosen as a location and  $X_{ij}$  be the quantity transported from plant at  $i$  to demand point  $j$ . Formulate the problem as an integer programming problem. You may consider the following:

The objective function minimizes the sum of the fixed costs of setting up the facilities and the costs of transporting the items. First constraint ensures that exactly  $p$  facilities are chosen. Next Constraint shall ensure that supply is possible only from a point chosen for setting up the facility and that the total amount transported from a chosen facility is less than the capacity. Remaining Constraint ensures that the demand constraints are met and the zero-one constraints are modelled. [15]

3. (a) State Bellman's Principle.  
 (b) A person wants to go from City 1 to City 10. The various possibilities and the distances are given below Table.

1-2	5	2-7	8	4-6	5	6-9	7
1-3	5	3-5	8	4-7	7	7-8	5
1-4	6	3-6	10	5-8	6	7-9	7
2-5	4	3-7	5	5-9	8	8-10	8
2-6	7	4-5	4	6-8	9	9-10	9

Obtain the minimal path from Source to Destination using Dynamic Programming. [15]



4. (a) The Dynaco Manufacturing Company produces a product in a process consisting of operations of five machines. The probability distribution of the number of machines that will break down in a week follow:

Machine Break-downs per week	Probability
0	.10
1	.10
2	.20
3	.25
4	.30
5	.05
	1.00

Simulate the machine breakdowns per week for 20 weeks. Compute the average number of machines that will break down per week. Use the following random numbers

21, 71, 12, 48, 18.08, 05, 18, 26, 94, 65, 18, 17, 89, 83, 90, 89, 08, 47, 06

- (b) Every time a machine breaks down at the Dynaco Manufacturing Company, either 1,2, or 3 hours are required to fix it, according to the following probability distribution:

Repair Time (hr.)	Probability
1	.30
2	.50
3	.20
	1.00

Simulate the repair time for 20 weeks using the same random numbers given in part a) and then compute the average weekly repair time.

If it costs \$50 per hour to repair a machine when it breaks down (including lost productivity), determine the average weekly breakdown cost.

[20]

5. The Bay city parks and Recreation Department has received a federal grant of \$600,000 to expand its public recreation facilities. City council representatives have demanded four different types of facilities-Gymnasiums, athletic fields, tennis courts, and swimming pools. In fact, the demand by various communities in the city has been for 7 gyms, 10 athletic fields, 8 tennis courts, and 12 swimming pools. Each facility costs a certain amount, requires a certain number of acres, and expected to be used a certain amount, as follows:

Facility	Cost	Required Acres	Expected Usage (people/week)
Gymnasium	\$ 80,000	4	1,500
Athletic field	24,000	5	3,000
Tennis court	15,000	3	500
Swimming pool	40,000	5	1,000

The park and recreation department has located 50 acres of land for construction (although more land could be located, if necessary). The department has established the following goals, listed in order of their priority:

- (a) The department wants to spend the total grant because any amount not spent must be returned to the government.
- (b) The department wants the facilities to be used by a total of at least 20,000 people each week.
- (c) the department wants to avoid having to secure more than the 50 acres of land already located.
- (d) The department would like to meet the demands of the city council for new facilities. However, this goal should be weighted according to the number of people expected to use each facility.
  - i. Formulate a goal programming model to determine how many of each type of facility should be constructed to best achieve the city's goals.

[15]

6. Bernard Mee, the head of the department of management science at Tech, is evaluating faculty for raises at the end of the academic year. He is considering three faculty members for raises: John Abbott, Megan Bates and Debbie Cook. Faculty evaluations are based on three criteria - teaching, research, and service. Professor Mee's pairwise comparisons for each of the three faculty members for each criterion and his pairwise comparison matrix for the three criteria are as follows:

Faculty Member	Teaching		
	A	B	C
A	1	2	1/3
B	1/2	1	1/5
C	3	5	1

Faculty Member	Research		
	A	B	C
A	1	3	1/2
B	1/3	1	1
C	2	1	1

Faculty Member	Service		
	A	B	C
A	1	3	6
B	1/3	1	2
C	1/6	1/2	1

Criterion	Teaching	Research	Service
A	1	3	5
B	1/3	1	2
C	1/5	1/2	1

Determine an overall ranking of the three faculty members by using AHP. Check the pairwise comparisons for the above criteria for consistency and indicate whether the level of consistency is acceptable.

The RI values for  $n$  items being compared is given below

n	2	3	4	5	6	7	8	9	10	
RI	0	0.58	0.90	1.12	1.24	1.31	1.41	1.45	1.51	[20]

7. (a) Describe the queuing model  $(M|M|1) : (50|FIFO)$ .
- (b) Trucks lineup at a highway weighbridge according to poisson process with mean arrival rate of 12 per hour. The time taken to service the truck follows an exponential distribution with average of 4 minute per truck. Find
- Percentage utilization of weighbridge.
  - Average time required for the truck to wait and get serviced.
  - Probability of finding exactly 6 trucks in queue.
  - Average number of trucks queuing up. [15]
8. Annual demand for the product is 50,000 units. Ordering and holding costs are Rs. 25 per order and Rs. 0.10 per unit per year. Determine the FOQ. If lead time is 20 days determine reorder level. [15]