

Indian Statistical Institute
SQC & OR Unit Bangalore
MS-QMS (Semester I): Operations Research I
Mid-Sem 2026-27
Time: 2 Hours Date: 09/09/2025

Answer all questions and score 30 marks; the value at the end of the question within the bracket represents the maximum marks; a scientific calculator is allowed.

Q1. Consider the following LPP:

$$\begin{aligned} \text{Maximize } Z &= 4x + 3y, \text{ subject to,} \\ 2x + y &\leq 8, \\ x + 2y &\geq 10, \\ 8x + 10y &\leq 80, \\ x, y &\geq 0. \end{aligned}$$

Do you find any redundant constraint in this problem and justify your answer? Then solve using the graphical method. [1+4=5]

Q2. A recycling center in Hyderabad manages three types of waste: Plastic, Paper, and Metal. There are five facilities, each with different costs and capacities. Additionally, each facility has a priority environmental score (higher score = more eco-friendly). The city generates 120 tons of waste per day: 60 tons of plastic, 40 tons of paper, and 20 tons of metal. The details of these facilities are given in the following table:

Facility	Transport Cost (INR/kg)	Processing Cost (INR/kg)	Max Capacity (tons)	Eco Score
1	0.6	5.5	40	8
2	0.9	4.2	50	6
3	1.1	3.8	60	9
4	0.7	6.0	30	7
5	0.5	4.5	35	10

Each facility can process all three types of waste, but at least 30% of plastic must go to facilities with eco score ≥ 8 . The paper must be processed in

at least two different facilities. Metal must not exceed 15 tons in any single facility. Formulate a mathematical model for this problem concerning the objective to be minimized with all constraints. Describe all steps clearly. (*do not solve*) [5]

Q3. Write the dual of the following problem and solve the dual problem with the help of the (suitable) simplex method. Generate the solution of the primal from the dual optimal solution.

$$\begin{aligned} \text{Maximize } Z &= 5x + 10y + 8z \quad \text{subject to,} \\ 3x + 5y + 2z &\leq 60, \\ 4x + 4y + 4z &\leq 72, \\ 2x + 4y + 5z &\leq 100, \\ x, y, z &\geq 0. \end{aligned}$$

Describe all the steps clearly. [2+5+2=9]

Q4. A dietician wants to minimize the cost while meeting the daily requirements for Carbohydrates, protein and Vitamins using two grains labeled 1 and 2. The table below provides information on the nutrient content of each grain and outlines the minimum daily requirements (MDR) for these nutrients.

Nutrient	Nutrient units/kg.		MDR in units
	Grain 1	Grain 2	
Carbohydrate	7	9	10
Protein	5	3	18
Vitamins	3	2	4
Cost (Rs/kg.)	70	45	

Table 1: Nutrient Units and Costs for Grain Types

Formulate the problem to minimize the cost and solve it using the **dual simplex method** for optimality. [2+5=7]

Q5. (i) Check the convexity of the set $S = S_1 \cap S_2$, where, $S_1 = \{\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2 \mid 1 \leq x_1 \leq 2, 0 \leq x_2 \leq 2\}$ and $S_2 = \{\mathbf{x} \in \mathbb{R}^2 \mid x = \lambda(1, 0) + (1 - \lambda)(2, 0), 0 < \lambda < 1\}$. [2]
(ii) Express $(5, 2)$ if possible as a CLC of points $(1, 2)$ and $(7, 1)$. [2]