Indian Statistical Institute
SQC & OR Unit Bangalore
MS-QMS (Semester I): Operations Research I
Mid-Sem - 2025-26
Time: 2 Hours Date: /09/2025

Answer all the questions and you can score 30 marks. The value at the end of the question within the bracket represents the maximum marks. A scientific calculator is allowed.

Q1. A recycling center in Bangalore has four distinct processing facilities to manage its plastic waste. Each of these facilities has a transportation and a processing cost to process a kilogram of plastic waste. The details of these facilities are given in the following table:

Facility No.	Transportation Cost (INR/kg)	Processing Cost (INR/kg)	Max daily capacity (in tons)
1	0.5	6	20
2	0.8	4	30
3	1	3	35
4	0.7	5	25

Table 1: Facility Costs and Capacities

Bangalore generates 70 tons of plastic waste per day. Formulate a mathematical model for this problem concerning the objective to be minimized with all bounding constraints. Describe all the steps clearly. [5]

Q2. Consider the following LPP:

$$\begin{aligned} &\text{Maximize} \quad Z = 2x + y, \quad \text{subject to,} \\ &x + y \leq 3, \\ &-3x + y \geq 3, \\ &x, y \geq 0. \end{aligned}$$

Solve using a graphical method, if applicable. Or else, suggest another method with explanation. [5]

Q3. Solve the following LPP with the help of the simplex method:

$$\begin{aligned}
&\text{Maximize } Z = 5x + 10y + 8z \quad \text{subject to,}\\
&3x + 5y + 2z \leq 60,\\
&4x + 4y + 4z \leq 72,\\
&2x + 4y + 5z \leq 100,\\
&x, y, z \geq 0.
\end{aligned}$$

Describe all the steps clearly. [6]

Q4. In a dietary chart, there are two grains labeled 1 and 2. These grains are composed of three nutrients: starch, protein, and vitamins. The table below provides information on the nutrient content of each grain and outlines the minimum daily requirements (MDR) for these nutrients.

Nutrient	Nutrient units/kg.		MDR
	Grain 1	Grain 2	in units
Starch	5	7	8
Protein	4	2	15
Vitamins	2	1	3
Cost (Rs/kg.)	60	35	

Table 2: Nutrient Units and Costs for Grain Types

Formulate the problem to minimize the cost and solve it using the two-phase method for optimality. [8]

Q5. (i) Check the convexity of the set $C = \{\mathbf{x} \in R^3 \mid x_2^2 + x_3^2 \leq 1, x_1 = 1\}$.[3]
(ii) Express $(0.3, 0.2)$ as a CLC of points $(0, 0), (2, 0)$ and $(1, 1)$. [3]