Indian Statistical Institute
SQC & OR Unit Bangalore
MS-QMS (Semester I): Operations Research I
End Semester: 2025-26
Time: 3 Hours Date: /11/2025

Answer all the questions and you can score 50 marks. The marks are given in parentheses. A scientific calculator is allowed.

Q1. You are given a task of selecting an investment portfolio for a client. You have selected a set of five alternatives with estimated values of the rate of return (annually) and risk as follows:

Alternatives	Rate of return (annual, in %)	Risk
Government bonds	6.0	1.3
Corporate bonds	8.0	1.5
Stocks	5.0	1.9
Mutual funds	7.0	1.7
Real estate	15.0	2.7

You must maximize the average annual rate of return in the portfolio. However, you must simultaneously ensure that the average risk of the portfolio is not greater than 2; and the investment in real estate should be less than 20%. Formulate a mathematical model for this problem. Describe all the steps clearly. [5]

Q2. The central dairy farm in Bangalore is concerned due to the high transportation cost incurred by daily orders to its retail centers located in different parts of the city. It has three dairy farms, say D1, D2, D3, and three retail centers, say R1, R2, R3. The per unit transportation cost from different farms to different retail centers is given in the following table:

	R_1	R_2	R_3	**Supply (units)**
D_1	5	10	10	55
D_2	20	30	20	80
D_3	15	20	30	75
Demand (units)	70	100	40	

(a) Model the problem as an LPP. [3]
(b) Find the initial BFS using the least cost method. [2]
(c) Find the optimal solution to the transportation problem using the $u - v$ method. [5]

Q3.A court administrator needs to assign four judges to four different dockets. The administrator has complete information regarding the court cases in each docket as well as the efficiency of each judge for dealing with different types of cases. The administrator has compiled the data into the following table, which shows the estimates of the number of court days each judge would require to completely process each court docket. The administrator would like to minimize the court days to completely process the court dockets by optimally assigning the judges.

Judges/Dockets	D_1	D_2	D_3	D_4
J_1	14	10	18	13
J_2	15	16	17	20
J_3	15	20	14	17
J_4	16	18	14	19

(a) Find the optimal assignment and the optimal assignment cost. [6]
(b) Reformulate this problem into equivalent transportation problem by constructing the appropriate parameter table. [2]
(c) Use the north-west corner method to obtain the initial BFS for the problem formulated in part(b). [2]

Q4.Write the dual of the following LPP and then solve the dual problem:

$$\begin{aligned}
&\text{Maximize } Z = 5x - 2y + 3z \\
&\text{subject to:} \\
&\quad 2x + 2y - z \geq 2, \\
&\quad 3x - 4y \leq 3, \\
&\quad x + 3y \leq 5, \\
&\quad x, y, z \geq 0.
\end{aligned}$$

Describe all the steps clearly and write the primal optimal solution. [10]

Q5. Solve the following LPP with the help of the Revised Simplex method:

$$\begin{aligned}
&\text{Maximize } Z = 2x + 4y \\
&\text{subject to:} \\
&\quad 2x + 3y \leq 48, \\
&\quad x + 3y \leq 42, \\
&\quad x, y \geq 0
\end{aligned}$$

Describe all the steps clearly. [10]

Q6. State and prove the weak duality theorem. [5]