SQC&OR Unit

Indian Statistical Institute, Bengaluru

Operations Research-I

Mid - Semester Examination

Time: 2 hrs 30 minutes Date: 13/09/2024

Answer as many questions as you can. Maximum score will be limited to 60

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1. Answer any five questions among the following: Maximum score (**5 ×2** =10)
   (1) Explain the use of slack, surplus, and artificial variables using an example.
   (2) Rewrite the following linear programming problem (LPP) in standard form.

   Min $z = 5x+7y$

   Subject to

   $2x+4y \geq -5$

   $5x+2y \leq 1$

   $x + y = 6$

   $x, y \geq 0$

   (3) Define an extreme point of a convex set and justify its application in LPP.
   (4) State and prove the minimax theorem on linear function and explain its application in LPP.
   (5) Modify the following LPP into its maximization canonical form.

   Min $z = 4x+3y$

   Subject to

   $x+2y= 3$

   $3x+4y \leq 1$

   $4x+y \geq 6$

   $x, y \geq 0$

   (6) What is the sufficient condition for the optimality of the simplex method, and when is a problem said to have a pseudo-optimal basic feasible solution?
2. Solve the following problem by simplex method: Maximum score (**1×5** =5)

   Min $Z=5x_1-6x_2+3x_3-5x_4+12x_5$

   Subject to

   $x_1+3x_2+5x_3+6x_4+3x_5 \leq 30.$

   $x_1, x_2, x_3, x_4, x_5 \geq 0.$
3. The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is $5 per unit on special risk insurance and $2 per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work hours per unit		Work hours available
	Special risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

   i) Formulate a linear programming model for this problem.
   ii) Use the graphical method to find the optimal solution for this model.
   iii) Verify the exact value of your optimal solution from part (*ii*) by solving the relevant two simultaneous equations algebraically.
   iv) Find the maximum expected profit. What are your suggestions to increase the profit?

   Maximum score (**2+4+2+3** =11)

4. Consider the following LPP: Maximum score (**1+6**=7)

   Min $z=3x+2y$

   Subject to

   $2x+y \leq 2$

   $3x+4y \geq 12$

   $x, y \geq 0$

   1) Reformulate this problem into the standard form of a linear programming problem.
   2) Use the Big M method to solve the above problem.

5. Consider the following LPP: Maximum score (**5+3+1** =9)

   Max $z=4x+3y$

   Subject to

   $4x+3y \leq 24$

   $x \leq 4.5$

   $y \leq 6$

   $x, y \geq 0$

   a. Use the graphical method to show that the optimal solution is a convex combination of two extreme points. Discuss the nature of the solution and explain the reason behind it.
   b. When the constraint $6x+2y \geq 12$, is added to the problem, what will be the optimal solution?
   c. If the problem is changed to a minimization problem with the addition of the constraint $6x+2y \geq 12$, what will be the new optimal solution?

6. Solve the following LPP using revised simplex method: maximum score (**1×5** =5)

   Max $z=4x+10y$

   Subject to

   $2x+3y \leq 90$

   $2x+5y \leq 100$

   $x, y \geq 0$

7. Solve the following LPP using Two-phase method: maximum score (**1×12** =12)

   Max $z=5x_1-4x_2+3x_3$

   Subject to

   $2x_1+x_2-6x_3= 20$

   $6x_1+5x_2+10x_3 \leq 76$

   $8x_1-3x_2+6x_3 \leq 50$

   $x_1, x_2, x_3 \geq 0$

8. In the Ma-and-Pa grocery store, shelf space is limited and must be used effectively to increase profit. Two cereal items, Grano and Wheatie, compete for a total shelf space of 60 ft^2. A box of Grano occupies 0.2 ft^2 and a box of Wheatie needs 0.4 ft^2. The maximum daily demands of Grano and Wheatie are 200 and 120 boxes, respectively. A box of Grano nets \$1.00 in profit and a box of Wheatie \$1.35. Ma-and-Pa thinks that because the unit profit of Wheatie is 35% higher than that of Grano, Wheatie should be allocated 35% more space than Grano, which amounts to allocating about 57% to Wheatie and 43% to Grano. What do you think?

   Maximum score (**1×6** =6)