Indian Statistical Institute, Bangalore

MS (QMS) First Year

First Semester - Operations Research I

Final Exam Date: November 18, 2024

Maximum marks: 50 Duration: 3 hours

Answer as many questions as you can. The maximum you can score is 50

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1. Cookwell, a manufacturer of large-size pressure cookers, produces and sells three models of cookers. While market demands pose no restriction, the capacity to produce is currently constrained by the limited supplies of special grade aluminium limited to 1,500 kg per week and machine processing time limited to 1,200 hours per week. In order to determine the optimal product-mix to maximize weekly contribution, a linear programming model as under was formulated:

$$\text{Max } Z = 60x_1 + 40x_2 + 80x_3$$

Subject to

$$6x_1 + 3x_2 + 5x_3 \leq 1500,$$
$$3x_1 + 4x_2 + 5x_3 \leq 1200,$$
$$x_1, x_2, x_3 \geq 0.$$

Using the simplex method, the following table was obtained:

| $C_B$ | $X_B$ | $X^0_B$ | $Y_1$ | $Y_2$ | $Y_3$ | $S_1$ | $S_2$ |
|---|---|---|---|---|---|---|---|
| * | $x_1$ | 100 | 1 | -1/3 | 0 | 1/3 | -1/3 |
| * | $x_3$ | 180 | 0 | 1 | 1 | -1/5 | 2/5 |
| | | $Z_j - C_j$ | * | * | * | * | * |

i) Fill in all the numerical values in starred (*) positions and check whether the current solution is optimal. If not, carry out the iterations until an optimal is reached. [3]

ii) Analyse the sensitivity of the optimal solution to the following changes, giving the new solution. (Each condition given below is independent and the analysis along with justification should be shown separately for each condition)

   a. Due to a machine breakdown, the machine hour available gets reduced to 1,050 hours. [2]

   b. An additional quantity of 50 kg of aluminium can be obtained. [2]

   c. The second model does not feature in the current optimal solution. What should be the minimum increase in unit contribution on this model for this to feature in the optimal solution? [1]

2. Consider the LPP:

$$\text{Max } Z=2x_1+4x_2+4x_3-3x_4$$

Subject to

$$x_1+x_2+x_3=4,$$

$$x_1+4x_2+x_4=8,$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

By using $x_3$ and $x_4$ as the starting variables, the optimum table is given by

| $C_B$ | $X_B$ | $X^0_B$ | $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ |
|---|---|---|---|---|---|---|
| 4 | $x_3$ | 2 | 3/4 | 0 | 1 | -1/4 |
| 4 | $x_2$ | 2 | 1/4 | 1 | 0 | 1/4 |
| | | $Z_j-C_j$ | 2 | 0 | 0 | 3 |

1. Write the dual problem and find its solution from the optimal primal table. [5]
2. State and prove the complementary slackness theorem. [5]
3. Verify that the theorem is valid for this problem. [5]

3. Use dual simplex method to solve the following LPP : [7]

$$\text{Minimize } Z= 3x_1+x_2 ,$$

Subject to

$$x_1+x_2 \geq 1,$$

$$2x_1+3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

4. A firm produces four products. There are four operators who are capable of producing any of these four products. The processing time varies from operator to operator. The firm records 8 hours a day and allows 30 minutes for lunch. The processing time in minutes and the profit for each of the product are given below:

| | Products | | | |
|---|---|---|---|---|
| Operators | A | B | C | D |
| 1 | 15 | 9 | 10 | 6 |
| 2 | 10 | 6 | 9 | 6 |
| 3 | 25 | 15 | 15 | 9 |
| 4 | 15 | 9 | 10 | 10 |
| Profit (Rs. per unit) | 8 | 6 | 5 | 4 |

1. Find the corresponding profit matrix. [4]
2. Determine the optimal assignment of product to operator. [6]

5. A manufacturer must produce a product in sufficient quantity to meet contractual sales in next four months. The production capacity and unit cost of production vary from month to month. The product produced in one month may be held for sale in later months but at an estimated storage cost of Re. 1 per unit per month. No storage cost is incurred for goods sold in the same month in which they are produced. There is no opening inventory and none is desired at the end of four months. The necessary details are given in the following table:

| Month | Contracted Sales | Maximum Production | Unit cost of production |
|---|---|---|---|
| 1 | 20 | 40 | 14 |
| 2 | 30 | 50 | 16 |
| 3 | 50 | 30 | 15 |
| 4 | 40 | 50 | 17 |

The manufacturer identified two optimal production plans to minimize the total cost. What are they? [10]

6. WELLDONE company has taken the third floor of a multistoried building for rent with a view to locate one of their zonal offices. There are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Some have windows; some are close to the washrooms or to the canteen or secretarial pool. The rooms are of all different sizes and shapes. Each of the five managers were asked to rank their room preferences among the rooms 301, 302, 303, 304 and 305. Their preferences were recorded in a table as indicated below:

| Manager 1 | Manager 2 | Manager 3 | Manager 4 | Manager 5 |
|---|---|---|---|---|
| 302 | 302 | 303 | 302 | 301 |
| 303 | 304 | 301 | 305 | 302 |
| 304 | 305 | 304 | 304 | 304 |
| | 301 | 305 | 303 | |
| | | 302 | | |

Most of the managers did not list all the rooms since they were not satisfied with some of these rooms and they have left off these from the list. Assuming that the preference can be quantified by numbers, find out as to which manager should be assigned to which room so that their total preference ranking is a minimum. [5]