Indian Statistical Institute, Bengaluru.

Operations Research-I

Time: 3 hour | *End-Term* | Maximum Marks: 50

Instruction: Attempt any 5 questions out 7.

1. A project has the following time (in weeks) and cost (in \$) and time duration of each activity:

Activity	Immediate predecessor	Normal time	Crash time	Normal cost	Crash Cost
A	-	4	3	11000	11700
B	A	3	1	7000	9000
C	A	2	1	5000	5600
D	B	4	3	14000	16000
E	B,C	1	1	2000	2000
F	C	3	2	8700	10000
G	E,F	4	2	23000	28000
H	D	3	1	10000	12200

The indirect cost for the project is \$800 per day.

(a) (3 points) Draw the network and find the time for the completion of the project.

(b) (3 points) Find the optimal cost by crashing the project and the completion time of the project. Indicate the activities that are crashed.

(c) (4 points) Find the minimum possible cost for the project if you want to finish it in 11 weeks. Indicate the activities that are crashed and how many days are crashed for each activity.

2. (10 points) A payoff matrix is given as following:

$$\begin{pmatrix} -4 & 2 & 5 & -6 & 6 \\ 3 & -9 & 7 & 4 & 8 \end{pmatrix}$$

Solve the game using the graphical method and find the strategies of two players and the value of the game.

3. (10 points) A linear programming problem is given as following:

$$Max\ Z \quad -5x_1 + 5x_2 + 13x_3$$

Subject to constraints

$$\begin{gathered} -x_1 + x_2 + 3x_3 \leq 20 \\ 12x_1 + 4x_2 + 10x_3 \leq 90 \\ x_1, x_2, x_3 \geq 0. \end{gathered}$$

(a) (3 points) Find the range of optimality for each of the objective coefficients.

(b) (2 points) Find range of feasibility for each constraint.

(c) (2 points) What happens when we put the coefficient of x_3 in the objective as $C_3 = 10$ or $C_3 = 20$? Explain.

(d) (3 points) What happens when we change the coefficients of x_2 and x_3 in the objective to $C_2 = 14/3$ and $C_3 = 29/2$? Explain.

4. (10 points) Use Dual Simplex method to solve the following LPP.

$$Minimize\ Z \quad x_1 + 2x_2 + 3x_3$$

Subject to constraints

$$\begin{aligned} x_1 - x_2 + x_3 &\geq 4 \\ x_1 + x_2 + 2x_3 &\leq 8 \\ x_2 - x_3 &\geq 2 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

5. (a) (5 points) A private company has manufacturing plants located in four cities A_1, A_2, A_3 and A_4. Its market demand necessitates warehouses in five cities B, C, D, K and S. Let the maximum capacities of the plants and the demand requirements of the warehouses be as shown in Table below. In this table, the number in each cell represents the cost of transporting a unit from the particular plant to the given warehouse. Find the least cost transportation program so that all the demands can be met. Then find the optimum solution to minimize the cost.

	B	C	D	K	S	Plant capacity
A_1	1	2	6	2	3	800
A_2	3	4	5	8	1	600
A_3	3	1	1	2	6	200
A_4	4	7	3	5	4	400
Warehouse demand	400	100	700	300	500	

(b) (5 points) Let we increase the capacity of the plant A_3 to 250 and the demand at the warehouse S to 550. Assume that the optimal basic set remains the same for the new problem. What happens to the transportation cost? Explain.

6. Consider the polyhedron K formed by the half spaces

$$x_2 \geq 3, \quad -x_1 + x_2 \leq 1, \quad -x_1 + 2x_2 \leq 5$$

(a) (2 points) Identify the extreme points and extreme direction of K.

(b) (3 points) Let x_1, x_2 be extreme points and d_1, d_2 be direction of K. Represent $(7/2, 4)$ as a convex combination of extreme points and non-negative combination of extreme directions.

(c) (5 points) Let $C \subset \mathbb{R}^n, D \subset \mathbb{R}^n, \lambda \in \mathbb{R}$. We define

$$\lambda C = \{x : x = \lambda c, c \in C\}$$
$$\text{and } C + D = \{x : x = c + d, c \in C, d \in D\}.$$

Show that

1. In general, intersection of any collection of convex sets is a convex set.
2. If C is convex, then λC is convex.
3. If C and D are convex sets, then $(C + D)$ is a convex set.

7. Consider the following problem:

$$Min\ c^t x$$

Subject to

$$\begin{gathered} Ax \geq b \\ x \geq 0 \end{gathered}$$

where A is an $m \times n$ matrix, c and x are $n \times 1$ matrices, and b is an $m \times 1$ matrix.

(a) (2.5 points) State duality theorem.

(b) (2.5 points) State the complementary slackness conditions.

(c) (5 points) Using part (a) and (b) above, verify that (2,2,4,0) is a solution to the following linear programming problem.
Maximize

$$2y_1 + 4y_2 + y_3 + y_4$$

Subject to

$$\begin{gathered} y_1 + 3y_2 + y_4 \leq 8 \\ 2y_1 + y_2 \leq 6 \\ y_2 + y_3 + y_4 \leq 6 \\ y_1 + y_2 + y_3 \leq 9 \\ \text{and } y_j \geq 0 \ (1 \leq j \leq 4) \end{gathered}$$