Indian Statistical Institute, Bangalore

MS (QMS) First Year

First Semester - Operations Research - I

Final Exam | Date: 05th January 2022
Maximum Marks: 60 | Duration: 3 hours

Answer as many as you can. Maximum you can score 60 marks.

1. Solve the following LPP. Also write its dual problem and solution. (10)

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$
Subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \geq 0$$

2. Consider the following LPP. (20)

Maximize $Z = -x_2 + 3x_3 - 2x_5$
Subject to

$$x_1 + 3x_2 - x_3 + 2x_5 = 7$$
$$-2x_2 + 4x_3 + x_4 = 12$$
$$-4x_2 + 3x_3 + 8x_5 + x_6 = 10$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

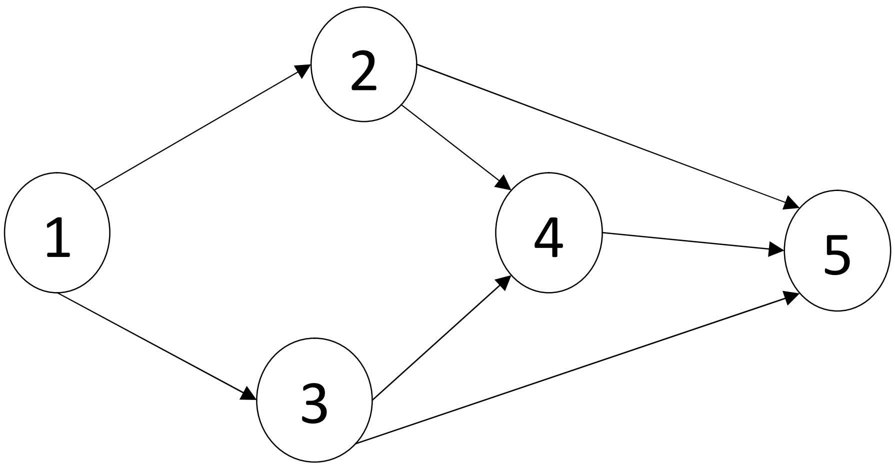
Iteration-4		C_j	0	-1	3	0	-2	0	
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	MinRatio $\frac{X_B}{x_6}$
x_2	-1	3.12	0.32	1	0	0.14	0	-0.08	---
x_3	3	4.56	0.16	0	1	0.32	0	-0.04	---
x_5	-2	1.1	0.1	0	0	-0.05	1	**(0.1)**	$\frac{1.1}{0.1} = 11$ →
$Z = 8.36$		Z_j	**-0.04**	**-1**	**3**	**0.92**	**-2**	**-0.24**	
		$Z_j - C_j$	-0.04	0	0	0.92	0	-0.24 ↑	

(a). Using the above iteration 4, find the optimal solution.
(b). Find the limits of variation of the costs C_1 *and* C_3 . Discuss when C_3 is taken as 3.

(c). Take RHS value 12 as b_2 . Determine the variation and discuss the optimality.

(d). Discuss the change in the co-efficient a_{24} *and* a_{23} and find the ranges within which the above change will lie so that the current solution will remain optimal.

3. Consider the following minimum cost network flow problem. **(20)**



$C_{13} = 6$
$C_{25} = 7$
$C_{24} = 5$
$C_{45} = 4$
$C_{35} = 3$
$C_{12} = 8$
$C_{34} = 6$

Nodes:	**1**	**2**	**3**	**4**	**5**
Availability:	**6**	**0**	**4**	**-5**	**-5**

(a). Discuss the minimum cost network flow problem.
(b). Formulate the problem as an LPP. Write down the DUAL of the LPP.
(c). Solve the network flow problem to obtain the minimum cost of flow.

4. Consider the following LPP. **(20)**

$$\textit{Maximize } Z = 4x_1 + 5x_2 + 9x_3 + 11x_4$$
Subject to
$$x_1 + x_2 + x_3 + x_4 \le 15$$
$$7x_1 + 5x_2 + 3x_3 + 2x_4 \le 3$$
$$3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100$$
$$x_1, x_2, x_3, x_4 \ge 0$$

(a). What variable do you enter into the basis at iteration 1of the simplex method if the objective function to be maximized is:

(i) $14x_1 + 5x_2 + 9x_3 + 11x_4$ (ii) $4x_1 + 5x_2 + 9x_3 + 8x_4$

(iii) $4x_1 + 5x_2 - 9x_3 + 11x_4$ (iv) $-4x_1 - 5x_2 - 9x_3 - 11x_4$

(b). Suppose you enter variable x_4 into the basis at iteration 1 of the simplex method, what variable do you remove from the basis and what will the new value of x_4 be if:

(i). The co-efficient on right-hand side of Row 2 is 24? Is 10?

(ii). The co-efficient on right-hand side of Row 3 is 330? Is 75?

(iii). The co-efficient of x_4 in Row 3 is 25? Is 10? Is -25?

(iv). The co-efficient of x_4 in Row 2 is 30?

(v). The co-efficient on the right-hand sides of Row 1, 2 and 3 are 44, 66, and 22, respectively, and the co-efficient of x_4 in these rows are -5, 10, and 2, respectively.

(c). Suppose you enter variable x_4 into the basis at iteration 1 of the simplex method. What are the new values of the basic variables if:

(i). you remove variable x_5 instead of x_7?

(ii). you remove variable x_6 instead of x_7?

(iii). Are the values in parts (i) and (ii) feasible? Explain.