Nonlinear Programming Mid Term Examination
MSQMS 2016
Indian Statistical Institute,Hyderabad
September 08, 2016

Answer all the questions.
Maximum Marks: 50

1. (a) Which of the following functions is convex, concave or neither? Why?
   - $f_1(x) = -|x|$
   - $f_2(x) = -x^{1/2}, x \geq 0$
   - $f_3(x, y) = e^x + e^y$
   - $f_4(x) = \log(x)$ [2x4]

   (b) Let $g : R \to R$ be a nondecreasing, univariate, convex function, and let $h : R^n \to R$ be a convex function. Then, prove that the composite function $f : R^n \to R$ defined as $f(x) = g[h(x)]$ is a convex function.   [5]

2. Consider the function $f(x, y) = x^2 + y^2$, where x and y are real numbers. Show that the set $S = \{(x, y, z) : z \geq f(x, y)\} \subseteq R^3$ is a convex set.   [5]

3. Consider the set $S = \{u : u = Ax, x \geq 0, u_3 = 2\}$, where

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 1 & 0 & -1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

   Is S a convex set? Is S a simplex?   [5]

4. Suppose you run the popular "Hotch Potch" ice cream outlet, which is open on all seven days of the week. Ice Cream is ordered from a manufacturer, who needs four days to prepare and fulfill the order. Ice Cream has a shelf life of three days. Based on past experience it is known that on some days demand for ice cream is more. The forecasted demand (in kilos) on different days (for a particular month) is:

$$\begin{bmatrix} \text{Day} & Sun & Mon & Tue & Wed & Thu & Fri & Sat \\ \text{Ice cream required} & 16 & 13 & 16 & 14 & 12 & 19 & 20 \end{bmatrix}$$

   The manufacturer can supply a maximum of 16 kilos of ice cream per order. Formulate the problem and determine the optimal decisions to be made in order to run the business profitably. Solve it using LINGO.   [30]