INDIAN STATISTICAL INSTITUTE
SQC & OR UNIT, HYDERABAD

MS in QUALITY MANAGEMENT SCIENCE 2015-17
III SEMESTER: FINAL EXAMINATION

Subject: Nonlinear Programming (NLP)
Date: 15-Nov-2016 Duration: 3 Hours Maximum Marks 60

INSTRUCTIONS

This paper carries 74 Marks. Answer as much as you can. Maximum you can score is 60.

1. State whether the following statements are true or false:

   a) If C is a closed convex cone of $\boldsymbol{R}^n$, then for every vector $x \in \boldsymbol{R}^n$, there exist $u \in C$ and $v \in C^*$ such that $x = u + v$.

   b) A function is convex if and only if its level sets are convex.

   c) Let $f(x, y) = x \sin y^2$. Then f is increasing in the direction $d = (1, \frac{\pi}{2})^t$ at $z = (1, \frac{\pi}{3})^t$.

   d) The direction $d = (-1, 2)^t$ for the problem: Minimize $f(x, y) = x \sin y^2$ is a feasible direction.

   e) Consider the problem:

   $$\begin{aligned} \texttt{Minimize} \quad & x^2 e^{-2x} + 3y^3 \\ \texttt{subject to} \quad & \\ & x + 2y \leq 4, \\ & x^2 - 3y = 28, \\ & 2x + y^2 \leq 12. \end{aligned}$$

   Then every direction is a feasible direction at the point $z = (5,\ -1)^t$.

   f) $f(x) = \ln(x - 3), x > 3$, is a quasiconvex function.

   g) The function $f(x) = \frac{a^t x + 3}{b^t x + 7}$, for $x \in \boldsymbol{R}^n$ such that $b^t x + 7 > 0$ is a convex function (a and b are fixed vectors in $\boldsymbol{R}^n$). $(7 \times 3 = 21)$.

2. Minimize $x_1^2 + 2x_2$
   Subject to

   $$(x_1 - 2)^2 + X_2^2 \leq 9$$

   take the initial point as (2,0).

a) Draw the feasible region.

b) Is (2,0) a local optimum?

c) Does the problem have an optimal solution? Justify your answer quoting a theorem.

d) Does this problem have an interior point optimal solution?

e) What are the extreme points of the feasible region?

f) Show that one of the extreme points is optimal.

g) Starting at (2,0) find the next point for exploring using the method of feasible directions. $(7 \times 4 = 28)$

3. Let f be a twice differentiable function defined on an open set $S \subseteq \boldsymbol{R}^n$. Suppose $z \in S$ is not local optimal. Find a feasible direction d in which the function decreases. (5)

4. (a) Define semidefinite programming problem and give a nontrivial but simple example. (5)

   (b) Consider the binary integer problem: Minimize $c^t x$ subject to $Ax \leq b$, x is a binary vector. Formulate the problem as a semidefinite programming problem. (5)

5. Suppose we have a refinery that must be ship finished goods to some storage tanks. Suppose further that there are two pipelines, A and B, to do the shipping. The cost of shipping x units on A is ax^2; the cost of shipping y units on B is by^2, where a and b are known positive numbers. How can we ship Q units while minimizing cost? What happens to the cost if Q increases by 5%. Formulate the problem. Write down KKT conditions. State whether they are necessary and sufficient for the problem. Solve the problem. (10)