Indian Statistical Institute

Semester Examination: 2025 – 2026

Master of Science in Quality Management Science, Semester II

Compulsory Optional: Neural Networks

Date: 05 May 2026 Maximum Marks: 100 Duration: 3 hours

Attempt all the questions. Credit will be given for precise and brief answers. Use diagrams wherever appropriate.

1. Define a perceptron. Let the perceptron has two dimensional input (x,y), where x and y can take binary values. Define the operation XOR (exclusive or) as ⊕, such that x⊕y = x'.y + x.y'. Show that the perceptron cannot classify the outcome of XOR operation on two dimensional binary values. 4 + 8 = 12
2. Consider a mixture of masoor dal and moong dal spread out in the thinnest layer on a plate, where each grain of each type of the dal has grain of the other type of the dal as its nearest neighbor. Represent this as a computer loadable data type. Define a mapping on this data to a higher dimensional space so that the two types of dal become linearly separable. Show that it is impossible to separate the two types of dal on the two dimensional plane by any supervised learning method. 2 + 4 + 6 = 12
3. Prove that eigenvalues of a real symmetric matrix will be all real and mutually orthogonal. Mention two drawbacks of PCA with very brief reasoning. 5 + 3 + 2 + 2 = 12
4. (a) Following is the weight matrix of a Hopfield network:

$$\mathbf{w} = \frac{1}{3}\begin{bmatrix} 0 & -2 & 2 \\ -2 & 0 & -2 \\ 2 & -2 & 0 \end{bmatrix}.$$

Please draw the diagram of the network indicating the nodes, direction and weight of the synapses. What will be the output of the network if the input is $(1,-1,1)$? From the result what can you infer about the input? 4 + 2 + 2 = 8

(b) What is Hebbian learning? Give the mathematical expression for it. 2 + 2 = 4

5. Describe shattering and then describe VC dimension in terms of shattering. What does the VC dimension say about a classifier? What is the VC dimension of the straight line on the Euclidean plane? Answer with justification. 4 + 3 + 2 + 3 = 12
6. Describe the architecture of a radial basis function network with a diagram. Describe a method to reduce the number of hidden layer nodes in the RBF network. Show by drawing a diagram how Gaussian RBFs approximate a continuous function. 5 + 3 + 4 = 12
7. Describe the principle of competitive learning. What is self-organizing mapping? Draw the architecture of self-organizing mapping network. Then explain the competitive learning in the self-organizing mapping network. 3 + 3 + 3 + 3 = 12

8. (a) Why Baye's classifiers are called naïve classifiers? Why are they optimum? 2 + 2 = 4
   (b) Describe max pooling with a diagram. 4
   (c) Write a short note on evolutionary computation. 4
   (d) In a multilayer perceptron why the error back propagation algorithm converges to a minimum? 4