INDIAN STATISTICAL INSTITUTE

MS in QMS

TEST ON MULTIVARIATE DATA ANALYSIS

Date: 27 February, 2026 Time: 2 hours Maximum Marks: 50

Answer as many questions as you can. The maximum you can score is 50

1. Show that for any random vector with finite second moments, its covariance matrix Σ is symmetric and positive semi definite?

[10]

2. Explain the Cholesky decomposition. Perform the Cholesky decomposition on the 3 x 3 symmetric positive definite matrix A:

$$A = \begin{bmatrix} 4.1. & 2.2 & 2.3 \\ 2.2 & 3.5 & 1.8 \\ 2.3 & 1.8 & 3.9 \end{bmatrix}$$

[15]

3. Suppose x is a multivariate normally distributed random vector with mean vector μ and covariance matrix Σ. Show that the maximum likelihood estimator of μ is the sample mean vector $\bar{x}$ and that the maximum likelihood estimator of Σ is sample covariance matrix ?

[10]

4.
   a. Let $x_1, x_2, - - -, x_n$ be *i.i.d* multivariate normally distributed random vectors with mean vector μ and covariance matrix Σ. Show that the sample mean vector

   $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

   is also multivariate normally distributed with mean vector μ and covariance matrix $\frac{1}{n}\Sigma$?

   b. Suppose $x \sim N_p(\mu, \Sigma)$. Show that Mahalanobis distance

   $$D^2 = (x-\mu)^T \Sigma^{-1} (x-\mu)$$

   is χ^2 distributed with p degrees of freedom?

[10]