Paper-6:

Elements of Maths 1 - End-Sem Question Paper

MS LIS First Year

November ?, 2015

Instructions: Answer all the questions. The maximum you can score is 60 marks. Marks corresponding to each question is indicated in bold. Maximum time allotted is 3 hrs.

(1) [3] Suppose $2+4i$ is a root of $x^2+bx+c=0$, where $b, c \in \mathbb{R}$. Find b and c.

(2) [7] Suppose that there are 100 identical chocolates and 5 kids. In how many ways can these chocolates be distributed among the kids such that each kid receives at least 1 chocolate and no two kids receive the same number of chocolates.

(3) [4+4] Suppose that there are n distinct pairs of shoes. Show that the number of ways (denote it by $f(n)$) in which the shoes can be paired such that no left shoe is paired with its correct right shoe

(a) satisfies the recurrence $f(n) = (n-1)[f(n-1) + f(n-2)]$ with $f(1) = 0$, $f(2) = 1$.

(b) and $f(n) = n!(\sum_{i=0}^{n} \frac{(-1)^i}{i!})$ for $n \geq 1$.

(4) [5] Use Binomial theorem or a combinatorial argument or otherwise prove the following identity $\sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i} = \binom{2n}{n}$

(5) [4] Compute $\sum_{i=0}^{n} i2^i$

(6) [3] Suppose that x, y and z are positive integers. Use AM-GM inequality or otherwise show that $(\frac{x}{y}+\frac{y}{z})(\frac{y}{z}+\frac{z}{x})(\frac{z}{x}+\frac{x}{y}) \geq 8$

(7) [4] Suppose C_1 is a circle with radius 1 unit centred at $(1,1)$. We recursively define C_i as the circle with smaller radius than that of C_{i-1} that touches C_{i-1} , X and Y axes for each $i \geq 2$. Compute the sum of areas of circles C_i

(8) [3+3] Prove that the circles $(x-1)^2+(y-2)^2 = 9$ and $(x-3)^2+(y-5)^2 = 1$ intersect. Find the equation of the straight line passing through the points of intersection of these circles.

(9) [3+3] Prove that the points $(1,2)$, $(3,4)$ and $(6,8)$ are non-collinear. Find the equation of the circle passing through these points.

(10) [4] How many circles of radius 3 units pass through $(1,2)$ and $(3,4)$? Justify. (Hint: The line segment joining the given points would be a chord if such a circle exists)

(11) [4] Find the farthest point on the circle $x^2+y^2=4$ to the point $(0,1)$. Justify your answer.

(12) [6] Let C be the circle centred at $(0, \sqrt{3})$ with radius 3 units. Prove that there do not exist more than two points on C with both their co-ordinates rational.