INDIAN STATISTICAL INSTITUTE
BANGALORE CENTRE

STUDENTS' BROCHURE

M.MATH. PROGRAMME
2011-12

8^{th} MILE MYSORE ROAD
BANGALORE 560 059

INDIAN STATISTICAL INSTITUTE
M.MATH. PROGRAMME

Contents

1 GENERAL INFORMATION

1.1 Scope

The M.Math. programme offers advance level training in Mathematics. On successful completion of the course, interested students would be able to pursue a research/academic career in Mathematics. Depending on the choice of the optional subjects, the students would also be able to work in the fields of Probability Theory and Theoretical Computer Science, as well as in private organizations.

1.2 Duration

The total duration of the M.Math. programme is four semesters. An academic year usually starts in July-August and continues till May, consisting of two semesters with a recess in-between. There is a study-break of one week before the semestral examinations in each semester. The time-table, prepared at the beginning of each semester, do not have an off day in the beginning or the end of the week.

1.3 Center

The programme is offered in Kolkata and Bengaluru in alternate years, with a batch commencing in Kolkata in even years and in Bengaluru in odd years.

1.4 Course Structure

The M.Math. programme comprises five courses in each of the four semesters. The courses are divided in two groups, one consisting of thirteen compulsory courses and the other, the optional courses, where seven have to be chosen out of a list of twenty-two courses. The courses, their sequencing and the syllabi are given later.

1.5 Examinations and Scores

The final (semestral) examination in a course is held at the end of the semester. Besides, there is a midsemestral examination in each course. The calendar for the semester is announced in advance. The midsemestral examinations are held over a maximum period of two weeks

The composite score in a course is a weighted average of the scores in the mid-semestral and semestral examinations, homework, assignments, and/or project work in that course; the weights are announced beforehand by the Dean of Studies, the In-Charge, Students' Academic Affairs or the Class Teacher, in consultation with the teacher concerned . The minimum composite score to pass a course is 35%.

If the composite score of a student falls short of 45% in a credit course, or 35% in a non-credit course, the student may take a back-paper examination to improve the score. At most one back-paper examination is allowed in each course. Moreover, a student can take at most four back-paper examinations (for credit courses) in the first year and only two (for credit courses) in the second year. The decision to allow a student to appear for the back-paper examination is taken by the appropriate Teachers' Committee. The back-paper examination covers the entire syllabus of the course. When a student takes back-paper examination in a credit course, his final score in that course is the higher of the back-paper score and the earlier composite score, subject to a maximum of 45%.

If a student gets less than 35% in at most one course after the back-paper examination, but gets 60% or more in average in other courses of that academic year excluding the course under consideration, the student can appear for a compensatory paper in the course under consideration. A student can appear in at most one compensatory paper every academic year. However, in the final year of the programme, the student can either appear in the compensatory paper, if the conditions stated above are met, or repeat the year if the existing rules so allow; and not do both. The student must inform the Dean of Studies or the In-Charge, Academic Affairs in writing in advance regarding his/her choice. No compensatory paper will be allowed in a course where backpaper is not allowed, e.g., Statistics Comprehensive in B. Stat. programme. The compensatory examinations for all subjects will be held once in an academic year. A student can score at most 35% in a compensatory paper. If a student scores more than 35% in a compensatory paper, the composite score in the course will be 35%. Any student who scores less than 35% in a compensatory paper will have to discontinue the programme regardless of the year of study in the academic programme.

There will be supplementary examination for mid-semestral, semestral, back-paper and compensatory examinations within a month of the examination missed by a student due to medical or family emergencies. The student should submit a written application to the Dean of Studies or the In-Charge, Academic Affairs for appearing in the supplementary examination, enclosing supporting documents. On receipt of such application from a student with supporting documents, the Dean of Studies or the In- Charge, Academic Affairs will decide, in consultation with the relevant Teachers' Committee, on whether such examination will be allowed. The student can score at most 60% in the supplementary examinations to mid-semestral and semestral examinations. For the back-paper or the compensatory papers, the maximum the student can score in the supplementary examination, is 45% or 35% respectively.

A student may take more than the allotted quota of backpaper examinations in a given academic year, and decide at the end of that academic year which of the backpaper examination scores should be disregarded.

1.6 Satisfactory Conduct

A student is also required to maintain satisfactory conduct as a necessary condition for taking semestral examination, for promotion and award of degree. Unsatisfactory conduct will include copying in examination, rowdyism, other breach of discipline of the Institute, unlawful/unethical behavior and the like. Violation of these is likely to attract punishments such as withholding promotion / award of degree, withdrawing stipend and/or expulsion from the hostel / Institute.

Ragging is banned in the Institute and any one found indulging in ragging will be given punishment such as expulsion from the Institute, or suspension from the Institute/classes for a limited period and fine. The punishment may also take the shape of (i) withholding Stipend/Fellowship or other benefits, (ii) withholding of results, (iii) suspension or expulsion from hostel and the likes. Local laws governing ragging are also applicable to the students of the Institute. Incidents of ragging may also be reported to the police.

The students are also required to follow the following guidelines during the examinations:

i. Students are required to take their seats according to the seating arrangement displayed. If any student takes a seat not allotted to him/her, he/she may be asked by the invigilator to hand over the answer script (i.e., discontinue the examination) and leave the examination hall.

ii. Students are not allowed to carry inside the examination hall any mobile phone with themeven in switched-off mode. Calculators, books and notes will be allowed inside the examination hall only if these are so allowed by the teacher(s) concerned (i.e., the teacher(s) of the course), or if the question paper is an open-note/open-book one. Even in such cases, these articles cannot be shared.

iii. No student is allowed to leave the examination hall without permission from the invigilator(s). Further, students cannot leave the examination hall during the first 30 minutes of any examination. Under no circumstances, two or more students writing the same paper can go outside together.

iv. Students should ensure that the main answer booklet and any extra loose sheet bear the signature of the invigilator with date. Any discrepancy should be brought to the notice of the invigilator immediately. Presence of any unsigned or undated sheet in the answer script will render it (i.e., the unsigned or undated sheet) to be cancelled, and this may lead to charges of violation of the examination rules.

v. **Any student caught cheating or violating examination rules for the first time will get Zero in that examination. If the first offence is in a backpaper examination, the student will get Zero in the backpaper.** (The other conditions for promotion, as mentioned in Section 1.7 of the Students Brochure will continue to hold).

vi. **If any student is caught cheating or violating examination rules for the second/third time and he/ she**

   (a) is in the final year of any programme and not already repeating, then he/she will have to repeat the final year without stipend;

   (b) is in the final year of any programme and already repeating, then he/she will have to discontinue the programme;

   (c) is not in the final year of any programme, then he/she will have to discontinue the programme even if he/she was not repeating that year.

   Any student caught cheating or violating examination rules for the second/third time, will be denied further admission to any programme of the Institute.

Failing to follow the examination guidelines, copying in the examination, rowdyism or some other breach of discipline or unlawful/unethical behavior etc. are regarded as **unsatisfactory conduct.**

The decisions regarding promotion in Section 1.7 and final result in Section 1.8 are arrived at taking the violation, if any, of the satisfactory conducts by the student, as described in this Section.

1.7 Promotion

A student is considered for promotion to the next year of the programme only when his/her conduct has been satisfactory. Subject to the above condition, a student is promoted from First Year to Second Year if the average composite score in all credit courses taken in the first year is not less than 45%, and no composite score in a course is less than 35%, and at most 4 courses with composite course less than 45%.

1.8 Final Result

At the end of the second year, the overall average of the percentage composite scores in all the credit courses taken in the two-year programme is computed for each student. The student is awarded the M.Math. degree in one of the following categories according to the criteria he/she satisfies, provided, in the second year, he/she does not have a composite score of less than 35% in a course, and his/her conduct is satisfactory.

Final Result	*Score*
M.Math., First Division with Distinction	(i) The overall average score is at least 75%, and (ii) the composite score in at most two credit courses is less than 45%.
M.Math., First Division	(i) Not in First Division with Distinction, (ii) the overall average score is at least 60%, and (iii) the composite score in at most four credit courses is less than 45%.
M.Math., Second Division	(i) Not in First Division with Distinction or First Division, (ii) the overall average score is at least 45%, and (iii) the composite score in at most four credit courses is less than 45%.

All other students are considered to have failed the programme.The students who fail but obtain at least 35% average score in the second year, have not taken any compensatory examination in the final year and have satisfactory conduct are allowed to repeat the final year of the M.Math. programme without stipend; the scores obtained during the repetition of the second year are taken as the final scores in the second year. A student is not given more than one chance to repeat the second year of the programme.

1.9 Award of Certificate

A student passing the M.Math. degree examination is given a certificate which includes (i) the list of all credit courses taken in the two-year programme along with the respective composite scores, (ii) the list of all non-credit courses passed and (iii) the category (First Division with Distinction or First Division or Second Division) of his/her final result.

The certificate is awarded in the Annual Convocation of the Institute following the last semestral examinations.

1.10 Prizes and Medals

Students are awarded prizes in form of book awards for good academic performances in each semester as decided by the Teachers Committee.

The best M.Math. student of the Institute, as decided by the Teachers Committee based on the academic performance, is given the ISI Alumni Association Mrs. M. R. Iyer Memorial gold medal.

1.11 Class-Teacher

One of the instructors of a class is designated as the Class Teacher. Students are required to meet their respective Class Teachers periodically to get their academic performance reviewed, and to discuss their problems regarding courses.

1.12 Attendance

Every student is expected to attend all the classes. If he/she is absent, he/she must apply for leave to the Dean of Studies or the In-Charge, Students' Academic Affairs. Failing to do so may result in disciplinary action.

1.13 Stipend

Stipend, if awarded at the time of admission, is valid initially for the first semester only. The amount of stipend to be awarded in each subsequent semester will depend on academic performance and conduct, as specified below, provided the requirements for continuation of the academic programme (excluding repetition) are satisfied; see Section 1.7.

Performance in course work:

All composite scores used in the following are considered after the respective back-paper examinations.

i. If all the requirements for continuation of the programme are satisfied, and the average composite score is at least 60% and the number of credit course scores less than 45% is at most one in any particular semester, the full value of the stipend is awarded in the following semester.

ii If all the requirements for continuation of the programme are satisfied, and the average composite score is at least 45% and the number of credit course scores less than 45most one in any particular semester, then half stipend is awarded in the following semester.

iii In all cases other than i. and ii. above, no stipend is awarded in the following semester.

Attendance

i. If the overall attendance in all courses in any particular semester is less than 75stipend is awarded in the following semester.

Conduct:

i. The Dean of Studies, the In-charge, Students' Academic Affairs or the Class Teacher, at any time, in consultation with the respective Teachers' Committee, may withdraw the stipend of a student fully for a specific period if his/her conduct in the campus is found to be unsatisfactory.

Note: The net amount of the stipend to be awarded is determined by simultaneous and concurrent application of all clauses described above; but, in no case, the amount of stipend to be awarded or to be withdrawn should exceed 100% of the prescribed amount of stipend.

Stipends can be restored because of improved performance, but no stipend is restored with retrospective effect.

Stipends are given after the end of each month for eleven months in each academic year. The first stipend is given two months after admission with retrospective effect provided the student continues in the M.Math. programme for at least two months.

Contingency grants can be used for purchasing a scientific calculator and other required accessories for the practical class, text books and supplementary text books and for getting photostat copies of required academic material. All such expenditure should be approved by the Class Teacher. No contingency grants are given in the first two months after admission.

1.14 Library Rules

Any student is allowed to use the reading room facilities in the library and allowed access to the stacks. M.Math. students have to pay a security deposit of Rs. 250 (in Kolkata) / Rs. 500 (in Bengaluru) in order to avail him/herself of the borrowing facility. A student can borrow at most four books at a time.

Any book from the Text Book Library (TBL) collection may be issued out to a student only for overnight or weekend provided at least one copy of that book is left in the TBL. Only one TBL book is issued at a time to a student. Fine is charged if any book is not returned by the due date stamped on the issue-slip. The library rules, and other details are posted in the library.

1.15 Placement

Students who have successfully completed the M.Math. programme are now well placed in government and semi-government departments, public and private sector undertakings, and industries/service organizations. Most of the students of the Institute get employment offers even before they complete the qualifying degree examinations.

There is a Placement Committee in Kolkata, which arranges campus interviews by prospective employers. Campus interviews are also organized at the Bengaluru centre.

1.16 Hostel Facilities

The Institute has hostels for male and female students in its premises in Kolkata and Bengaluru. However, it may not be possible to accommodate all students in the hostels. Limited medical facilities are available free of cost at Kolkata and Bengaluru campuses. Students, selected for stay in the hostels, have to pay Rs. 605 (in Kolkata) / Rs. 800 (in Bengaluru) as hostel deposit, whereas the hostel rent of Rs. 60 is deducted from their monthly stipend.

The Institute campus in Kolkata is about 12 km from the city centre. The Bengaluru campus is about 20 km. from the respective city centres.

1.17 Change of Rules

The Institute reserves the right to make changes in the above rules, course structure and the syllabi as and when needed.

2 DETAILED COURSE STRUCTURE

2.1 M. Math. Curriculum

First Year

Semester I	**Semester II**
1.Analysis of Several Variables	1.Complex Analysis
2.Topology I	2.Functional Analysis
3.Linear Algebra	3.Algebra II
4.Algebra I	4.Topology II
5.Measure Theoretic Probability	5.Differential Geometry I

Second Year

Semester I	**Semester II**
1.Basic Probability Theory/Elective	1.Elective
2.Fourier Analysis	2.Elective
3.Differential Topology	3.Elective
4.Elective	4.Elective
5.Elective	5.Elective

2.2 List of Compulsory Courses

1. Analysis of Several Variables
2. Topology I
3. Linear Algebra
4. Algebra I
5. Measure Theoretic Probability
6. Complex Analysis
7. Functional Analysis
8. Algebra II

9. Topology II
10. Differential Geometry I
11. Basic Probability Theory (for non-BMath non-BStat students)
12. Fourier Analysis
13. Differential Topology

2.3 List of Elective Courses

Group A *(At least one elective course is to be chosen from this group.)*

1. Number Theory
2. Advanced Number Theory
3. Algebraic Number Theory

Group B

1. Differential Equations
2. Graph Theory and Combinatorics
3. Advanced Functional Analysis
4. Operator Theory
5. Partial Differential Equations
6. Advanced Linear Algebra
7. Advanced Probability
8. Markov Chains
9. Ergodic Theory
10. Stochastic Processes
11. Topology III

12. Topology IV
13. Differential Geometry II
14. Algebra III
15. Commutative Algebra I
16. Commutative Algebra II
17. Algebraic Geometry (PRQ: Op 1)
18. Elliptic Curves
19. Representations of Locally Compact Groups
20. Lie Groups & Lie Algebra
21. Linear Algebraic Groups
22. Mathematical Logic
23. Set Theory
24. Game Theory
25. Automata, Languages and Computation
26. Advanced Fluid Dynamic
27. Quantum Mechanics I
28. Quantum Mechanics II
29. Analytical Mechanics
30. Special Topics (to be suggested by the faculty)
31. Projects I and II

3 BRIEF SYLLABI

3.1 Compulsory Courses

Analysis of Several Variables

Differentiability of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ and the derivative as a linear map. Higher derivatives, Chain Rule, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier. Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e., product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustrated with plenty of examples. Inverse and implicit functions theorems. Picard's Theorem.

Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$. Line integrals, Surfaces in $\mathbb{R}^3$, Surface integrals, Integration of forms, Divergence, Gradient and Curl operations, Green's, Strokes' and Gauss' (Divergence) theorems.

Suggested Texts:

1. M. Spivak, Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus, Benjamin/Cummings (1965).
2. W. Rudin, Principles of Mathematical Analysis, Mc Graw-Hill Ed Asia (1953).
3. T. Apostol, Mathematical Analysis, Narosa Pub House (2002).
4. R. Courant and F. John, Introduction to Calculus and Analysis Vol II, Springer (1989).
5. T. Apostol, Calculus (Vol 2), Wiley Eastern (1980).

Topology

Topological spaces, open and closed sets, basis, closure, interior and boundary. Subspace topology, Hausdorff spaces. Continuous maps: properties and constructions; Pasting Lemma. Homeomorphisms. Product topology, Quotient topology and examples of Topological Manifolds. Connected, path-connected and locally connected spaces. Lindelof and Compact spaces, Locally compact spaces, one-point compactification and Tychonoff's theorem. Paracompactness and Partitions of unity. Countability and separation axioms. Urysohn's lemma, Tietze extension theorem and applications. Completion of metric spaces. Baire Category Theorem and applications. Time permitting, Urysohn embedding lemma and metrization theorem for second countable spaces.

Covering spaces, Path Lifting and Homotopy Lifting Theorems, Fundamental Group.

Suggest Texts:

1. J. R. Munkres, Topology: a first course, Prentice-Hall of India (2000).
2. K. Janich, Topology, UTM, Springer (Indian reprint 2006).
3. M.A. Armstrong, Basic Topology, Springer (Indian reprint 2004).
4. G.F. Simmons, Introduction to Topology and Modern Analysis, TataMcGraw- Hill (1963).
5. J.L. Kelley, General Topology, Springer (Indian reprint 2005).
6. I. M. Singer and J. A. Thorpe, Lecture Notes on Elementary Topology and Geometry, UTM, Springer (Indian reprint 2004).
7. J. Dugundji, Topology, UBS (1999).

Linear Algebra

1. Review of linear transformations and matrices. Eigenvectors, characteristic polynomial, orthogonal matrices and rotations. Inner product spaces, Hermitian, unitary and normal transformations, spectral theorems, bilinear and quadratic forms. Multilinear forms, wedge and alternating forms.
2. Review of basic concepts from rings and ideals required for module theory (if necessary). Modules over commutative rings: examples. Basic concepts: submodules, quotients modules, homomorphisms, isomorphism theorems, generators, annihiliators, torsion, direct product and sum, direct summand, free modules, finitely generated modules, exact and split exact sequences. Time permitting: snake's lemma, complexes and homology sequences
3. Properties of $K[X]$ over a field K. Structure theorem for finitely generated modules over a PID; applications to Abelian groups, rational and/or Jordan canonical forms.

Suggested Texts:

1. D.S. Dummit and R.M. Foote, Abstract Algebra, John Wiley (Asian reprint 2003).
2. S. Lang, Algebra, GTM (211), Springer (Indian reprint 2002).
3. K. Hoffman and R. Kunze, Linear Algebra, Prentice-Hall of India (1998).
4. N.S. Gopalakrishnan, University Algebra, Wiley Eastern (1986).

Algebra I

1. Commutative rings with unity: examples, ring homomorphisms, ideals, quotients, isomorphism theorems with applications to non-trivial examples. Prime and maximal ideals, Zorn's Lemma and existence of maximal ideals. Product of rings, ideals in a finite product, Chinese Remainder Theorem. Prime and maximal ideals in a quotient ring and a finite product. Field of fractions of an integral domain. Irreducible and prime elements; PID and UFD.
2. Polynomial Ring: universal property; division algorithm; roots of polynomials. Gauss' Theorem (R UFD implies R[X] UFD); irreducibility criteria. Symmetric polynomials: Newton's Theorem. Power Series.
3. Noetherian rings and modules, algebras, finitely generated algebras, Hilbert Basis Theorem. Tensor product of modules: definition, basic properties and elementary computations. Time permitting, introduction to projective modules.
4. Groups: Review of normal subgroups, quotient groups and homomorhism theorems. Group actions with examples, class equations and their applications, Sylow's Theorems; groups and symmetry. Direct sum and free Abelian groups. Time permitting: composition series, exact sequences, direct product and semidirect product with examples.

Note: It is desirable that Item No. 1 of Algebra I is covered before Item No. 2 of Linear Algebra begins.

Suggested Texts:

1. D.S. Dummit and R.M. Foote, Abstract Algebra, John Wiley (Asian reprint 2003).
2. N. Jacobson, Basic Algebra Vol. I, W.H. Freeman and Co (1985).
3. S. Lang, Algebra, GTM (211), Springer (Indian reprint 2004).

4. N.S. Gopalakrishnan, University Algebra, Wiley Eastern (1986).
5. N.S. Gopalakrishnan, Commutative Algebra (chapter 1), Oxonian Press (1984).
6. J.J. Rotman, An Introduction to the theory of groups, GTM (148), Springer- Verlag (2002).

Measure Theoretic Probability

Measure and Integration: σ-algebras of sets, Monotone Class Theorem, Probability and σ-finite Measures, Construction of Lebesgue measure, Integration, Fatou Lemma, Monotone and Dominated Convergence Theorems, Radon- Nikodym theorem, product measures, Fubinis theorem.

Probability: (If needed, a quick review of concepts and results (without proof) from basic Discrete and Continuous Probabilty.) Distribution Functions of Probabilty Measures on $\mathbb{R}$, Borel-Cantelli Lemma, Weak and Strong Laws of Large Numbers in i.i.d. case, various Modes of Convergende, Characteristic Functions, Uniqueness/Inversion/Levy Continuity Theorems, Proof of the Central Limit Theorem for i.i.d. case with Finite Variance.

Suggested Texts:

1. W. Rudin, Real and complex analysis, McGraw-Hill Book Co. (1987).
2. P. Billingsley, Probability and measure, John Wiley (1995).
3. K. R. Parthasarathy, Introduction to probability and measure, TRIM (33), Hindustan Book Agency (2005).
4. J. Nevue, Mathematical foundations of the calculus of probability, Holden- Day (1965).
5. I. K. Rana, An introduction to measure and integration, Narosa Publishing House (1997).

Complex Analysis

A review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates. power series expansions, maximum modulus principle, Classification of singularities and calculus of residues. Normal families, Arzela's theorem. Product developments, functions with prescribed zeroes and poles, Hadamard's theorem. Conformal mappings, the Riemann mapping theorem, the linear fractional transformations.

Depending on time available, some of the following topics may be done:

(i) Subharmonic functions, the Dirichlet problem and Green's functions.

(ii) An introduction to elliptic functions.

(iii) Introduction to functions of several complex variables.

Suggested Texts:

1. L. V. Ahlfors, Complex analysis. An introduction to the theory of analytic functions of one complex variable, McGraw-Hill (1978).

2. J. B. Conway, Functions of one complex variable, GTM (159), Springer- Verlag (1995).

3. W. Rudin, Real and complex analysis, McGraw-Hill (1987).

4. R. Narasimhan and Y. Nievergelt, Complex Analysis in One Variable, Birkhauser (2001).

Functional Analysis

Normed linear spaces, Banach spaces. Bounded linear operators. Dual of a normed linear space. Hahn-Banach theorem, uniform boundedness principle, open mapping theorem, closed graph theorem. Computing the dual of wellknown Banach spaces. Weak and weak* topologies, Banach-Alaoglu Theorem. The double dual.

L^p spaces, Riesz representation theorem for the space $C[0,1]$.

Hilbert spaces, adjoint operators, self-adjoint and normal operators, spectrum, spectral radius, analysis of the spectrum of a compact operator on a Banach space, spectral theorem for bounded self-adjoint operators.

Time permitting: reflexivity; spectral theorem for normal and unitary operators.

Suggested Texts:

1. W. Rudin, Real and complex analysis, McGraw-Hill (1987).

2. W. Rudin, Functional analysis, McGraw-Hill (1991).

3. J. B. Conway, A course in functional analysis, GTM (96), Springer (Indian reprint 2006).

4. K. Yosida, Functional analysis, Springer (Indian reprint 2004).

Algebra II

Results on finite groups: permutation groups, simple groups, solvable groups, simplicity of A_n. Topics like semi-direct product (if not covered in Algebra-I).

Algebraic and transcendental extensions; algebraic closure; splitting fields and normal extensions; separable, inseparable and purely inseparable extensions; finite fields.

Galois extensions and Galois groups, Fundamental theorem of Galois theory, cyclic extensions, solvability by radicals, constructibility of regular n-gons, cyclotomic extensions.

Time permitting, additional topics may be selected from:

(i) Traces and norms, Hilbert theorem 90, Artin-Schrier theorem, Galois cohomology, Kummer extension.

(ii) Transcendental extensions; Luroth's theorem.

(iii) Real fields: ordered fields, real closed fields, Sturms theorem, real zeros and homomorphisms.

(iv) Integral extensions and the Nullstellensatz.

Suggested Texts:

1. D.S. Dummit and R.M. Foote, Abstract Algebra, John Wiley (Asian reprint 2003).
2. S. Lang, Algebra, GTM (211), Springer (Indian reprint 2004).
3. M. Nagata, Field theory, Marcel-Dekker (1977).
4. N.S. Gopalakrishnan, University Algebra, Wiley Eastern (1986).
5. N. Jacobson, Basic Algebra, W.H. Freeman and Co (1985).
6. G. Rotman, Galois theory, Springer (Indian reprint 2005).
7. TIFR pamphlet on Galois theory.

Topology II

1. Review of fundamental groups, necessary introduction to free product of groups, Van Kampen's theorem. Covering spaces, lifting properties, Universal cover, classification of covering spaces, Deck transformations, properly discontinuous action, covering manifolds, examples.

2. Categories and functors. Singular homology groups, axiomatic properties, Mayer-Vietoris sequence, homology with coefficients, statement of universal coefficient theorem for homology, simple computation of homolgy groups.

3. CW-complexes and Cellular homology, Simplicial complex and simplicial homology as a special case of Cellular homology, Relationship between fundamental group and first homology group.

Suggested Texts:

1. A. Hatcher, Algebraic Topology, Cambridge University Press (2002).

2. W. S. Massey, A Basic Course in Algebraic Topology, GTM (127), Springer (Indian reprint 2007).

3. J. R. Munkres, Elements of algebraic topology, Addison-Wesley (1984).

4. M. J. Greenberg and J.R. Harper, Algebraic topology: A First Course, Benjamin/ Cummings (1981).

5. I. M. Singer and J. A. Thorpe, Lecture Notes on Elementary Topology and Geometry, UTM, Springer (Indian reprint 2004).

6. E. Spanier, Algebraic Topology, Springer-Verlag (1982).

Differential Geometry I

Parametrized curves in $\mathbb{R}^3$, length of curves, integral formula for smooth curves, regular curves, parametrization by arc length. Osculating plane of a space curve, Frenet frame, Frnet formula, curvatures, invariance under isometry and reparametrization. Discussion of the cases for plane curves, rotation number of a closed curve, osculating circle, 'Umlaufsatz'.

Smooth vector fields on an open subset of $\mathbb{R}^3$, gradient vector field of a smooth function, vector field along a smooth curve, integral curve of a vectorfield. Existence theorem of an integral curve of a vector field through a point, maximal integral curve through a point.

Level sets, examples of surfaces in $\mathbb{R}^3$. Tangent spaces at a point. Vector fields on surfaces. Existence theorem of integral curve of a smooth vector field on a surface through a point. Existence of a normal vector of a connected surface. Orientation, Gauss map. The notion of geodesic on a surface. The existence and uniqueness of geodesic on a surface through a given point and with a given velocity vector thereof. Covariant derivative of a smooth vector field. Parallel vector field along a curve. Existence and uniqueness theorem of a parallel vector field along a curve with a given initial vector. The Weingarten map of a surface at a point, its self-adjointness property.

Normal curvature of a surafce at a point in a given direction. Principal curvatures, first and second fundamental forms, Gauss curvature and mean curvature. Surface area and volume. Surfaces with boundary, local and global stokes theorem. Gauss-Bonnet theorem.

Suggested Texts:

1. B. ONeill, Elementary Differential Geometry, Academic Press (1997).
2. A. Pressley, Elementary Differential Geometry, Springer (Indian reprint 2004).
3. J.A. Thorpe, Elementary topics in Differential Geometry, Springer (Indian reprint 2004).

Basic Probability Theory (Only for non B.Stat./B.Math. students)

Orientation. Combinatorial probability. Fluctuations in Coin Tossing and Random Walks. Combination of Events, Occupancy and Matching Problems. Conditional probabilities. Urn Models. Independence.

Random Variables, discrete distributions, Expectation, variance and moments, probability generating functions and moment generating functions, Tchebychevs inequality. Standard discrete distributions: uniform, binomial, Poisson, geometric, hypergeometric, negative binomial. Continuous random variables: univariate densities and distributions, Expectations, variance and moments, standard univariate densities: normal, exponential, gamma, beta, chi-square, Cauchy.

Joint and conditional distributions, Independence of random variables, Transformation of variables.

Laws of Large Numbers (proofs optional).

Suggested Texts:

1. S. Ross, First course in probability theory, Mac Millan (1989).

2. P.G. Hoel, S.C. Port and C.J. Stone, Introduction to Probability Theory, Universal Book Stall, New Delhi (1991).
3. W.Feller, An introduction to probability theory and its applications Vol 1, Wiley (1950).
4. K.L. Chung, Elementary Probability Theory, Springer (Indian reprint 2003).

Fourier Analysis

Fourier and Fourier-Stieltjes' series, summability kernels, convergence tests. Fourier transforms, the Schwartz space, Fourier Inversion and Plancherel theorem. Maximal functions and boundedness of Hilbert transform. Paley-Wiener Theorem. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem. Introduction to wavelets and multi-resolution analysis.

Suggested Texts:

1. E. M. Stein and R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press (2003).
2. Y. Katznelson, An introduction to harmonic analysis, Dover Publications (1976).
3. E.M. Stein and G.Weiss, Introduction to Fourier Analysis on Euclidean Spaces, Princeton University Press (1971).
4. E. Hernandez and G. Weiss, A first course on wavelets, Studies in Advanced Mathematics. CRC Press (1996).

Differential Topology

1. Manifolds in RN, submanifolds, smooth maps of manifolds, derivatives and tangents, Inverse function theorem and immersions, submersions, Transversality, Homotopy and stability, Sard's theorem and Morse functions, embedding manifolds in Euclidean space.
2. Differential Forms and Integration of forms, Stokes Theorem, Definition of de Rham Cohomology.

Suggested Texts:

1. V. Guillemin and A. Pollack, Differential Topology, Prentice-Hall (1974).

2. J.W. Milnor, Topology from the Differentiable Viewpoint, Princeton Univ. Press (1997).
   1. V. Guillemin and A. Pollack, Differential Topology, Prentice- Hall.

3. J.W. Milnor, Topology from the Differentiable Viewpoint, Univ Press of Virginia (1965).

3.2 Elective Courses

Group A

(A student must choose at least one elective from this group.)

Number Theory

1. Review of unique factorization; properties of the rings $Z[i]$ and $Z[\omega]$ (chapter 1 of IR).

2. Review of congruences, Euler's ϕ-function, results of Fermat, Euler and Wilson; linear congruences, Chinese remainder theorem. Primitive roots and the group structure of $U(Z/nZ)$; applications to congruences of higher degree; Hensels Lemma (chapter 4 of IR and sections 2.1 to 2.7 of NZM).

3. Quadratic Reciprocity: Quadratic Residues, Gaussian reciprocity law, the Jacobi symbol (chapter 5 of IR).

4. Arithmetic Functions, Moebius inversion formula and combinatorial methods like principle of inclusion-exclusion and pigeonhole etc (sections 4.2,4.3,4.5 of NZM).

5. Diophantine equations. Linear equations, the equation $x^2 + y^2 = z^2$. Method of Descent; the equation $x^4 + y^4 = z^2$ (section 5.1 to 5.4 of NZM).

6. Binary Quadratic forms. Sum of two squares. Legendre's Theorem (section 3.4 to 3.7 of NZM).

7. Simple continued fractions. Infinite continued fractions and irrational numbers. Periodic continued fractions, algorithms for solving Brahmagupta-Pell equation, numerical computations. Dirichlet's box principle and solution of Pell's equation (chapter 7 of NZM).

8. Elementary results on the function $\pi(x)$, Bertrands postulate (sections 8.1, 8.2 of NZM).

Time permitting, additional topics may be chosen from :

(i) Partitions. Euler's identity and Euler's formula (sections 10.1 to 10.4 of NZM).

(ii) Gauss and Jacobi Sums, Cubic and Biquadratic Reciprocity (from chapters 8,9 of IR).

(iii) Irrational numbers: Hurwitzs theorem on rational approximations; irrationality of certain values of trigonometric functions; irrationality of π(chapter 6 of NZM).

(iv) Diophantine equations over finite fields (chapter 10 of IR).

(v) Introduction to Zeta function, Dirichlet's L-functions and Elliptic Curves (from IR).

Suggested Texts:

1. (IR) K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory Second Edition, Springer (Indian reprint 2004).
2. (NZM) I. Niven, H.S. Zuckerman and H. Montgomery, An Introduction to the Theory of Numbers, John Wiley (1991); Indian edition available.
3. J.H. Silverman, A Friendly Introduction to Number Theory, Prentice-Hall (2005).
4. J. Stillwell, Mathematics and Its History Second Edition, Springer (Indian reprint 2004).

Advanced Number Theory

Review of finite fields; Polynomial equations over finite fields: theorems of Chevalley and Warning; Quadratic Forms over prime fields. Review of the law of quadratic reciprocity.

The ring of p-adic integers; the field of p-adic numbers; completion; padic equations and Hensel's lemma; Quadratic Forms with p-adic coefficients. Hilberts symbol.

Dirichlet series: abscissa of convergence and absolute convergence. Riemann Zeta function and Dirichlet L-functions. Dirichlet's theorem on primes in arithmetic progression. Functional equation and Euler product for L-functions.

Modular forms and the modular group $SL(2,\mathbb{R})$. Eisenstein series. Zeros and poles of modular functions. Dimensions of the spaces of modular forms. The j-invariant and Picards Theorem. L-function and Ramanujan's τ -function.

Suggested Texts:

1. J.P. Serre: A Course in Arithmetic, Springer-Verlag (1973).

2. Z. Borevich and I. Shafarevich: Number Theory (chapter 1), Academic Press (1966).
3. K. Chandrasekharan: Introduction to Analytic Number Theory, Springer- Verlag (1968).

Algebraic Number Theory

Review of norm and trace, Number fields and their rings of integers, Prime decomposition in number rings, Kummer-Dedekind discriminant criterion for ramification, The Ideal Class Group, ray class group, their finiteness and Dirichlet's Unit theorem, Valuations and completions of number fields, Decomposition and inertia groups, Frobenius automorphism, Artin symbol, Dedekind zeta function and the Distribution of ideals in a number ring, Kronecker limit formula, Frobenius density theorem. Time permitting, introduction to class field theory.

Suggested Texts:

1. G.J. Janusz: Algebraic Number Fields (chapter 1-4), AMS (1996).
2. D.A. Marcus: Number Fields, Springer-Verlag (1977).
3. J.Neukirch: Algebraic Number Theory, Springer (1999).
4. I. Stewart and D. Tall: Algebraic Number Theory and Fermat's Last Theorem, A.K. Peters (2001).
5. J. Esmonde and M. Ram Murty: Problems in Algebraic Number Theory, Springer (Indian reprint 2006).
6. TIFR pamphlet on Algebraic Number Theory.

Note: For students opting for "Algebraic Number Theory", a prior knowledge of "Commutative Algebra" is desirable.

Group B

Differential Equations
(Only for non BStat/BMath students)

Ordinary differential equations first order equations, Picard's theorem (existence and uniqueness of solution to first order ordinary differential equation), Second order linear equations second order linear differential equations with constant co-efficients, Systems of first order differential equations, Equations with regular singular points, Introduction to power series and power series solutions, Special ordinary differential equations arising in physics and some special functions (eg. Bessel's functions, Legendre polynomials, Gamma functions). Partial differential equations elements of partial differential equations and the three equations of physics i.e. Laplace, Wave and the Heat equations, at least in 2-dimensions. Lagranges method of solving first order quasi linear equations.

Suggested Text:

1. G.F. Simmons, Differential Equations with Applications and Historical Notes, McGraw-Hill (1994).

Note: The course may be combined with the B.Math./B.Stat. course on "Differential Equations". B.Math./B.Stat. degree holders cannot opt for this course.

Graph Theory and Combinatorics

A course based on either of the following sequence of topics may be offered.

- A Construction and Uniqueness of Finite Fields, Linear Codes, Macwilliams identity, Finite projective planes, strongly regular graphs and regular 2-graphs. t-designs with emphasis on Mathieu designs. Counting arguments and inclusionexclusion principle. Ramsey Theory: graphical and geometric.
- B B.Graphs and digraphs, connectedness, trees, degree sequences, connectivity, Eulerian and Hamiltonian graphs, matchings and SDR's, chromatic numbers and chromatic index, planarity, covering numbers, flows in networks, enumeration, inclusion-exclusion, Ramsey's theorem, recurrence relations and generating functions. Time permitting, some of the following topics may be done: (i) strongly regular graphs, root systems, and classification of graphs with least eigenvalue, (ii) Elements of coding theory (Macwilliams identity, BCH, Golay and Goppa codes, relations with designs).

Note: The course may also be combined with the "Graph Theory and Combinatorics" course of M.Stat. 2nd year.

Suggested Texts:

1. F. Harary, Graph Theory, Addision-Wesley (1969); Narosa (1988).

2. D.B. West, Introduction to Graph Theory, Prentice-Hall (1996); Indian ed (1999).

3. J.A. Bondy and U.S.R. Murty, Graph Theory with applications, Macmillan (1976).

4. H.J. Ryser, Combinatorial Mathematics, Carus Math Monographs; Math Assoc of America (1963).

5. M.J. Erickson, Introduction to Combinatorics, John Wiley (1996).

6. L. Lovasz, Combinatorial Problems and Exercises, AMS Chelsea (1979).

Advanced Functional Analysis

Brief introduction to topological vector spaces (TVS) and locally convex TVS. Linear Operators. Uniform Boundedness Principle. Geometric Hahn-Banach theorem and applications (Markov-Kakutani fixed point theorem, Haar Measure on locally compact abelian groups, Liapounovs theorem). Extreme points and Krein-Milman theorem. In addition, one of the following topics:

(a) Geometry of Banach spaces: vector measures, Radon-Nikodym Property and geometric equivalents. Choquet theory. Weak compactness and Eberlein- Smulian Theorem. Schauder Basis.

(b) Banach algebras,spectral radius, maximal ideal space, Gelfand transform.

(c) Unbounded operators, Domains, Graphs, Adjoints, spectral theorem.

Suggested Texts:

1. N. Dunford and J. T. Schwartz, Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space, Interscience Publishers, John Wiley (1963).

2. Walter Rudin, Functional analysis Second edition, International Series in Pure and Applied Mathematics. McGraw-Hill (1991).

3. K. Yosida, Functional analysis, Springer (Indian reprint 2004).

4. J. Diestel and J. J. Uhl, Jr., Vector measures, Mathematical Surveys (15), AMS (1977).

Operator theory

1. Compact operators on Hilbert Spaces: Fredholm Theory, Index.

2. C*-algebras - noncommutative states and representations, Gelfand-Neumark representation theorem.
3. Von-Neumann Algebras; Projections, Double Commutant theorem, L^∞ functional Calculus.
4. Toeplitz operators.

Suggested Texts:

1. W. Arveson, An invitation to C*-algebras, GTM (39), Springer-Verlag (1976).
2. N. Dunford and J. T. Schwartz, Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space, Interscience Publishers John Wiley (1963).
3. R. V. Kadison and J. R. Ringrose, Fundamentals of the theory of operator algebras. Vol. I. Elementary theory, Pure and Applied Mathematics (100), Academic Press (1983).
4. V. S. Sunder, An invitation to von Neumann algebras, Universitext, Springer- Verlag (1987).

Partial Differential Equations

Theory of Schwartz distributions and Sobolev spaces; local solvability and Lewys example; existence of fundamental solutions for constant coefficient differential operators; Laplace, heat and wave equations, hypoelliptic and analytic hypoelliptic operators, elliptic boundary value problems interior regularity, local existence.

Suggested Texts:

1. G. B. Folland, Introduction to partial differential equations, Princeton University Press (1995); reprinted Prentice Hall of India.
2. F. Trs, Basic linear partial differential equations, Pure and Applied Mathematics (62), Academic Press (1975).
3. J. Rauch, Partial differential equations, GTM (128), Springer-Verlag (1991).
4. E. DiBenedetto, Partial differential equations, Birkhauser (1995).
5. L. C. Evans, Partial differential equations, Graduate Studies in Mathematics (19), AMS (1998).

6. L. Hormander, The analysis of linear partial differential operators. I. Distribution theory and Fourier analysis., Springer-Verlag (1990).

Advanced Probability

Independence, Kolmogorov Zero-one Law, Kolmogorov Three-series theorem, Strong law of large Numbers. LevyCramer Continuity theorem, CLT for i.i.d. components, Infinite Products of probability measures, Kolmogorovs Consistency theorem, Radon-Nikodym Theorem, Conditional expectations.

Discrete parameter martingales with applications

Suggested Texts:

1. J. Nevue, Mathematical foundations of the calculus of probability, Holden- Day (1965).
2. P. Billingsley, Probability and measure, John Wiley (1995).
3. Y. S. Chow and H. Teicher, Probability theory. Independence, interchangeability, martingales, Springer Texts in Statistics, Springer (Indian reprint 2004).

Note: The syllabus for the "Advanced Probability" course of M.Stat. 2nd year may also be used.

Markov Chains

Finite State Markov Chains. Examples, Classification of States, Stationary Distribution.

Rates of convergence to stationarity, Dirichlet Form and Spectral gap methods, Some Coupling methods with applications, Random walk on Finite Groups, Poisson Processes, Continuous time Markov Chains, Birth-and-death processes.

Suggested Texts:

1. S. M. Ross, Stochastic processes, John Wiley (1996).
2. R. N. Bhattacharya and E. C. Waymire, Stochastic processes with applications.
3. E. GinG, R. Grimmett and L. Saloff-Coste, Lectures on probability theory and statistics, Springer-Verlag (1997).

Ergodic Theory

1. Measure preserving systems; examples. Hamiltonian dynamics and Liouville's theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skewproduct.
2. Poincare Recurrence lemma. Induced transformation: Kakutani towers: Rokhlins lemma. Recurrence in Topological Dynamics, Birkhoff's Recurrence theorem.
3. Ergodicity, Weak-mixing and strong-mixing and their characterisations. Ergodic theorems of Birkhoff and Von Neumann. Consequences of the ergodic theorems. Invariant measures on compact systems. Unique ergodicity and equidistribution. Weyl's theorem.
4. Isomorphism problem; conjugacy, spectral equivalence.
5. Transformations with discrete spectrum, Halmosvon Neumann theorem.
6. Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. Shannon- McMillan-Breiman Theorem.
7. Flows. Birkhoff's Ergodic Theorem and Wiener's Ergodic Theorem for flows. Flows built under a function.

Suggested Texts:

1. Peter Walters, An introduction to Ergodic Theory, GTM (79), Springer (Indian reprint 2005).
2. Patrick Billingsley, Ergodic theory and information, Robert E. Krieger Publishing Co. (1978).
3. M. G. Nadkarni, Basic ergodic theory, TRIM 6, Hindustan Book Agency (1995).
4. H. Furstenberg, Recurrence in ergodic theory and combinatorial number theory, Princeton University Press (1981).
5. K. Petersen, Ergodic theory, Cambridge Studies in Advanced Mathematics (2), Cambridge University Press (1989).

Stochastic Processes

Weak Convergence of probability measures on Polish spaces including C[0, 1], Brownian motion; construction, simple properties of paths, Connections between Brownian Motion / Diffusion and PDEs. Time permitting: Stationary processes, Markov processes and generators.

Suggested Texts:

1. P. Billingsley, Convergence of probability measures, John Wiley (1999).
2. K. Ito, Lectures on Stochastic Processes, TIFR Lecture Notes (1960).
3. D. Revuz andM. Yor, Continuous martingales and Brownian motion, Springer- Verlag (1999).
4. I. Karatzas and S.E. Shreva, Brownian Motion and Stochastic Calculus, GTM 113, Springer (1991).

Note:

(1) For students opting for this elective, a prior knowledge of the contents of the courses Probability I,II, III of the B.Stat. programme (ISI) and Advanced Probability of M.Stat./M.Math is essential.

(2) The course may be combined with the course "Stochastic Processes-I" of M.Stat. 2nd year.

Topology III

1. CW-complexes, cellular homology, comparison with singular theory, computation of homology of projective spaces.
2. Definition of singular cohomology, axiomatic properties, statement of universal coefficient theorem for cohomology. Betti numbers and Euler characterisitic. Cup and cap product, Poincare duality. Cross product and statement of Kunneth theorem. Degree of maps with applications to spheres.
3. Definition of higher homotopy groups, homotopy exact sequence of a pair. Definition of fibration, examples of fibrations, homotopy exact sequence of a fibration, its application to computation of homotopy groups. Hurewicz homomorphism, The Hurewicz theorem. The Whitehead Theorem.

Suggested Texts:

1. A. Hatcher, Algebraic Topology, Cambridge University Press (2002).
2. M. J. Greenberg and J.R. Harper, Algebraic topology: A First Course, Benjamin/ Cummings (1981).
3. E. Spanier, Algebraic Topology, Springer-Verlag (1982).
4. J.W. Vick, Homology Theory: an introduction to algebraic topology, Springer (1994).
5. J. R. Munkres, Elements of algebraic topology, Addison-Wesley (1984).
6. G.E. Bredon, Topology and Geometry, Springer (Indian reprint 2005).

Topology IV

1. Smooth manifolds, Differential forms on manifolds, Integration on manifolds, Stoke's theorem, computation of cohomology rings of projective spaces, Borsuk- Ulam theorem.
2. Degree, linking number and index of vector fields, The Poincare-Hopf theorem.
3. Definition and examples of principal bundles and fibre bundles, clutching construction, description of classification theorem (without proof).

Suggested Texts:

1. R. Bott and L. W. Tu, Differential forms in algebraic topology, GTM (82), Springer-Verlag (1982).
2. Ib H. Madsen and J. Tornehave, From Calculus to Cohomology: De Rham Cohomology and Characteristic Classes, Cambridge Univ Press (1997).
3. F. W. Warner, Foundations of differentiable manifolds and Lie groups, GTM (94), Springer-Verlag (1983).
4. D. Husemoller, Fibre Bundles, Springer-Verlag (1993).
5. N. Steenrod, The Topology of Fibre Bundles, Princeton Univ Press (1999).

Differential Geometry II

1. A quick review of tensors, alternating forms, manifolds, immersion, submersion and submanifolds.
2. Tangent bundle, vector bundles, vector fields, flows and the fundamental theorem of ODE. Embedding in Euclidean space, tubular neighbourhood. Differential forms and integration, Stoke's theorem. Transversality, Riemann metrics, Riemannian connection on Riemannian manifolds, Gauss-Bonnet theorem. Parallel transport, geodesics and geodesic completeness, the theorem of Hopf- Rinow.

Suggested Texts:

1. F. W. Warner, Foundations of differentiable manifolds and Lie groups, GTM (94), Springer-Verlag (1983).
2. S. Helgason, Differential geometry, Lie groups, and symmetric space, Graduate Studies in Mathematics (34), AMS (2001).
3. W.M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press (1975); Elsevier (2008).
4. J.M. Lee, Riemannian Manifolds: An Introduction to Curvature, GTM (176), Springer (1997).

Algebra III

1. Modules over noncommutative rings: Noetherian and Artinian rings and modules. Modules of finite length. Krull-Schmidt theorem.
2. Balanced maps, Tensor product of modules and algebras: definitions, basic properties, right exactness, change of base.
3. Semisimple rings and modules; Wedderburns structure theorem.
4. . Nilradical and Jacobson radical; NAK lemma; Jacobson radical of an Artinian ring is nilpotent; Ring semi-simple if and only if Artinian with trivial radical; Artinian ring is Noetherian.
5. Central Simple Algebras and the Brauer Group; Examples.
6. Representation of finite groups: group algebra, Maschkes Theorem, Simple Modules over Group Algebras; Characters and Orthogonality relations; Burnside's two-prime theorem; Induced representation; Frobenius reciprocity; Brauer's theorem on induced characters.

Suggested Texts:

1. T.Y. Lam, A First Course in Noncommutative Rings, Springer (2001).
2. C.W. Curtis and I. Reiner, Representation Theory of finite Groups and Associative Algebras, AMS Chelsea (1962).
3. P.M. Cohn, Further Algebra and Applications, Springer (Indian reprint 2004).
4. S. Lang, Algebra, Springer (Indian reprint 2004).
5. D.S. Dummit and R.M. Foote, Abstract Algebra (Part VI), JohnWiley (Asian reprint 2003).
6. TIFR notes on semisimple rings and modules.

Commutative Algebra

1. Rings and ideals: review of ideals in quotient rings; prime and maximal ideals, prime ideals under quotient, existence of maximal ideals; operations on ideals (sum, product, quotient and radical); Chinese Remainder theorem; nilradical and Jacobson radical; extension and contraction of ideals under ring homomorphisms; prime avoidance.
2. Free modules; Projective Modules; Tensor Product of Modules and Algebras; Flat, Faithfully Flat and Finitely Presented Modules; Shanuel's Lemma.
3. Localisation and local rings, universal property of localisation, extended and contracted ideals and prime ideals under localisation, localisation and quotients, exacteness property. Results on prime ideals like theorems of Cohen and Isaac. Nagata's criterion for UFD and applications; equivalence of PID and one-dimensional UFD.
4. Modules over local rings. Cayley-Hamilton, NAK lemma and applications. Examples of local-global principles. Projective and locally free modules. Patching up of Localisation.
5. Polynomial and Power Series Rings. Noetherian Rings andModules. Hilberts Basis Theorem. Associated Primes and Primary Decomposition. Artininan Modules. Modules of Finite Length.
6. Integral Extensions: integral closure, normalisation and normal rings. Cohen- Seidenberg Going-Up Theorem. Hilbert's Nullstellensatz and applications.
7. Valuations, Discrete Valuation Rings, Dedekind domains.

Suggested Texts:

1. N.S. Gopalakrishnan, Commutative Algebra, Oxonian Press (1984).
2. M.F. Atiyah and I.G. Macdonald, Introduction to commutative algebra, Addision-Wesley (1969).
3. M. Reid: Undergraduate commutative algebra, LMS Student Texts (29), Cambridge Univ. Press (1995).
4. R.Y. Sharp: Steps in commutative algebra, LMS Student Texts (19), Cambridge Univ. Press (1995).
5. E. Kunz: Introduction to commutative algebra and algebraic geometry, Birkhauser (1985).
6. D.S. Dummit and R.M. Foote: Abstract Algebra (Part V), John Wiley (Asian reprint 2003).
7. D. Eisenbud: Commutative algebra with a view toward algebraic geometry GTM (150), Springer-Verlag (1995).
8. F. Ischebeck and Ravi A. Rao, Ideals and Reality, Springer (2005).

Commutative Algebra II

1. Graded Rings and Modules. Artin-Rees Lemma. I-adic filtrations. Completion. Exacteness and Flatness properties. Krulls Intersection Theorem. Hensel's Lemma and applications. Weierstrass Preparation theorem.
2. Associated Primes. Primary Decomposition. Homomorphisms and AssM. SuppM.
3. Going-up and Going-Down Theorems. Finiteness of Integral Closure. Krull- Akizuki theorem.
4. Dimension Theory: Hilbert Samuel Polynomial. Krull's Principal Ideal Theorem. Dimension Theorem.
5. Normalisation Lemma. Hilberts Nullstellensatz. Integral Closure of Affine Domains.
6. Valuations, Discrete Valuation Rings, Dedekind Domains, Fractional and Invertible Ideals, Ramification Formula.

7. Results on Normal and Regular Rings: Local property of Normal Domains, Normality and DVR at height one primes, Intersection of DVRs; Jacobian criterion for regular local rings (of affine algebras).
8. Homological Algebra: Projective Resolution. The functors Ext and Tor. Homological Dimension. Injective Modules and Injective Resolution. Injective Dimension and Global Dimension. Global Dimension of Noetherian Local Rings.
9. Properties of Regular Local Rings. Homological Characterisation of Regular Local Rings. Regular Local Ring is a UFD.
10. Derivations and Modules of Differentials.

Suggested Texts:

1. N.S. Gopalakrishnan, Commutative Algebra, Oxonian Press (1984).
2. H. Matsumura, Commutative Algebra, W.A. Benjamin (1970).
3. TIFR pamphlet on Homological Methods in Commutative Algebra.
4. D. Eisenbud, Commutative algebra with a view toward algebraic geometry, GTM (150), Springer-Verlag (1995).
5. J.P. Serre, Local Algebra, Springer-Verlag (2000).
6. N. Bourbaki, Commutative Algebra, Springer-Verlag (1995).
7. M. Reid, Undergraduate commutative algebra, LMS Student Texts (29), Cambridge Univ. Press (1995).

Note: Prior knowledge of the content of the course "Commutative Algebra" is essential.

Algebraic Geometry

Topics from: Polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert Nullstellensatz, Affine and Projective spaces, Affine Schemes, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezout's theorem, Abelian differential, Riemann Roch for curves.

Suggested Texts:

1. W. Fulton, Algebraic curves. An introduction to algebraic geometry, Addison- Wesley (1989).
2. D.S. Dummit and R.M. Foote, Abstract Algebra (Part V), John Wiley (Asian reprint 2003).
3. C.G. Gibson, Elementary Geometry of Algebraic Curves, Cambridge (1999).
4. I.R. Shafarevich, Basic algebraic geometry, Springer (1994).
5. J. Harris, Algebraic geometry. A first course, GTM (133), Springer-Verlag (1995).
6. K. Kendig, Elementary algebraic geometry, GTM (44), Springer-Verlag (1977).
7. D. Mumford, The Red Book of Varieties and Schemes, Springer (1999).
8. C. Musili, Algebraic geometry for beginners, TRIM (20), HBA (2001).

Note: For students opting for "Algebraic Geometry", a prior knowledge of Commutative Algebra is desirable.

Elliptic Curves

Pre-requisites: a course in complex analysis, a course in number theory, a course in algebraic number theory (could be simultaneous), a course in algebraic geometry (could be simultaneous).

Syllabus:

1. Algebraic curves, divisors, Riemann-Roch theorem.
2. Definition of elliptic curves, Weierstrass form, isogeny, Tate module, Weil pairing, Endomorphism ring.
3. Elliptic functions and integrals, Elliptic curves over complex numbers, Uniformization.
4. Elliptic curves over finite fields, Weil conjectures, Hasse invariant.
5. Elliptic curves over local fields, Minimal Weierstrass equation, Torsion, Good and bad reduction and Neron-Ogg-Shafarevich criterion for good reduction.
6. Elliptic curves over global fields, weak Mordell-Weil, Kummer pairing, Mordell- Weil theorem over Q.

   If time permits : Heights on projective spaces and elliptic curves and Mordell-Weil theorem; Nagell-Lutz theorem.

Suggested Text:

J. Silverman, Arithmetic of elliptic curves (chapters 2,3,5,6,7 and sections 8.1 to 8.4), GTM 106, Springer-Verlag (1986).

Representations of Locally Compact Groups

Topological Groups, basic properties like subgroups, quotients and products, fundamental systems of neighbourhoods, open subgroups, connectedness and compactness. Existence of Haar measure on locally compact groups, properties of Haar measures.

Group actions on topological spaces, the space X/G in the topological as also in the analytical case assuming regularity conditions of the group action.

Compact groups: Unitarity of finite dimensional representations, Peter-Weyl theory, Representations of $SU(2,\mathbb{C})$, Representation of a finite group.

Induced representation and Frobenius reciprocity theorem, Representations of Heisenberg groups and of Euclidean motion group, Principal series representations of $SL(2,\mathbb{R})$.

Suggested Texts:

1. P. J. Higgins, Introduction to topological groups, LMS Lect Note Series (15), Cambridge University Press (1974).
2. L. H. Loomis: An introduction to abstract harmonic analysis, D. Van Nostrand (1953).
3. G. B. Folland: A course in abstract harmonic analysis, Studies in Advanced Mathematics. CRC Press (1995).

Note: Prior familiarity with "Representations of finite groups" (item 5 of "Algebra III") will be helpful.

Lie Groups and Lie Algebras

1. Linear Lie groups: the exponential map and the Lie algebra of linear Lie group, some calculus on a linear Lie group, invariant differential operators, finite dimensional representations of a linear Lie group and its Lie algebra. Examples of linear Lie group and their Lie algebras, e.g., Complex groups: $GL(n,\mathbb{C})$, $SL(n,\mathbb{C})$, $SO(n,\mathbb{C})$, Groups of real

matrices in those complex groups: $GL(n, \mathbb{R})$, $SL(n, \mathbb{R})$, $SO(n, \mathbb{R})$, Isometry groups of Hermitian forms SO(m, n),U(m, n), SU(m, n). Finite dimensional representations of $su(2)$ and $SU(2)$ and their connection. Exhaustion using the lie algebra $su(2)$. [2 weeks]

2. Lie algebras in general, Nilpotent, solvable, semisimple Lie algebra, ideals, Killing form, Lies and Engels theorem. Universal enveloping algebra and Poincare-Birkhoff-Witt Theorem (without proof). [6 weeks]

3. Semisimple Lie algebra and structure theory: Definition of Linear reductive and linear semisimple groups. Examples of Linear connected semisimple/ reductive Lie groups along with their Lie algebras (look back at 2 above and find out which are reductive/semisimple). Cartan involution and its differential at identity; Cartan decomposition $g = k + p$, examples of k and p for the groups discussed above. Definition of simple and semisimple Lie algebras and their relation, Cartans criterion for semisimplicity. Statements and examples of Global Cartan decomposition, Root space decomposition; Iwasawa decomposition; Bruhat decomposition. [6 weeks]

If time permits, one of the following topics:

(i) A brief introduction to Harmonic Analysis on $SL(2, \mathbb{R})$.

(ii) Representations of Compact Lie Groups and Weyl Character Formula.

(iii) Representations of Nilpotent Lie Groups.

Suggested Texts:

1. J.E. Humphreys: Introduction to Lie algebras and representation theory, GTM (9), Springer-Verlag (1972).

2. S.C. Bagchi, S. Madan, A. Sitaram and U.B. Tiwari: A first course on representation theory and linear Lie groups, University Press (2000).

3. Serge Lang: $SL(2, \mathbb{R})$. GTM (105), Springer (1998).

4. W. Knapp: Representation theory of semisimple groups. An overview based on examples, Princeton Mathematical Series (36), Princeton University Press (2001).

5. B.C. Hall, Lie Groups, Lie Algebras and Representations: An Elementary Introduction, Springer (Indian reprint 2004).

Linear Algebraic Groups

Pre-requisites: a course in commutative algebra, a course in Lie algebras (could be simultaneous), a course in algebraic geometry (could be simultaneous).

Syllabus: Review of background commutative algebra and algebraic geometry (as in chapter 1 of Humphreys's book or chapter 1 of Springer's book). Definition of affine algebraic groups and homomorphisms over algebraically closed fields, examples. Orbit-closures under actions, linearity of affine groups. Lie algebra of an algebraic group and adjoint representation. Homogeneous spaces and quotients, Chevalley's theorem. Correspondence between groups and Lie algebras. Jordan decomposition, Commutative linear algebraic groups, diagonalizable groups and algebraic tori. Definition of weights and roots, Weyl group. Unipotent groups, Lie-Kolchin theorem, Structure theorem for connected solvable groups. Definition of reductive and semisimple groups, Borel subgroups, parabolic subgroups. Basic facts on complete varieties, Borels fixed point theorem. Conjugacy of maximal tori, Nilpotency of Cartan subgroups. Density theorem and connectedness of centralizers of tori. Normalizer theorem for parabolic subgroups. Regular and singular tori, Structure theorem for groups of semi-simple rank one. Structure theorem for reductive groups, Bruhat decomposition, semisimple groups. Tits system, standard parabolic subgroups, simplicity proof. If time permits, mention (without proof): Representations and classification of semisimple groups and statements for general fields.

Suggested Texts:

1. J. E. Humphreys, Linear algebraic groups, (chapters 1 to 10), GTM, Springer- Verlag (1975).
2. T. A. Springer, Linear algebraic groups, (chapters 1 to 8), Progress in Mathematics, Birkhuser (1998).
3. R. Steinberg, Conjugacy classes in algebraic groups, (Chapters 1 and 2 just as reference, some proofs are not given), Lecture Notes in Mathematics (366), Springer-Verlag (1974).

Suggested Texts:

1. P. J. Higgins, Introduction to topological groups, LMS Lect Note Series (15), Cambridge University Press (1974).
2. L. H. Loomis: An introduction to abstract harmonic analysis, D. Van Nostrand (1953).
3. G. B. Folland: A course in abstract harmonic analysis, Studies in Advanced Mathematics. CRC Press (1995).

Mathematical Logic

Syntax of First-Order Logic: First Order Languages, Terms and Formulas of a First Order language, First Order Theories.

Semantics of First-Order Languages: Structures of First-Order Languages, Truth in a Structure, Model of a Theory.

Propositional Logic: Tautologies and Theorems of propositional Logic, Tautology Theorem.

Proof in First Order Logic, Metatheorems of a first order theory, e.g. , theorems on constants, equivalence theorem, deduction and variant theorems etc., Consistency and Completeness, Lindenbaum Theorem.

Henkin Extension, Completeness theorem, Extensions by definition of first order theories, Interpretation theorem.

Model Theory: Embeddings and Isomorphisms, Löwenheim-Skolem Theorem, Compactness theorem, Categoricity, Complete Theories. Recursive functions, Arithmatization of first order theories, Decidable Theory, Representability, Godel's first Incompleteness theorem.

Suggested Texts:

1. S. M. Srivastava, A Course on Mathematical Logic, Universitext, Springer (2008).

2. J. R. Shoenfield, Mathematical logic, Addison-Wesley (1967).

Set Theory

Either (A) or (B).

(A) Descriptive Set Theory.

1. A quick review of elementary cardinal and ordinal numbers, transfinite induction, induction on trees, Idempotence of Souslin operation.

2. Polish spaces, Baire category theorem, Transfer theorems, Standard Borel spaces, Borel isomorphism theorem, Sets with Baire property, Kuratowski-Ulam Theorem. The projective hierarchy and and its closure properties.

3. Analytic and coanalytic sets and their regularity properties, separation and reduction theorems, Von Neumann and Kuratowski-Ryll Nardzewskis selection theorems, Uniformization of Borel sets with large and small sections. Kondos uniformization theorem.

Suggested Texts:

1. S. M. Srivastava, A course on Borel sets, GTM (180), Springer-Verlag (1998).
2. A. S. Kechris, Classical descriptive set theory, GTM (156), Springer-Verlag (1995).

(B) Axiomatic Set Theory

1. A naive review of cardinal and ordinal numbers including regular and singular cardinals, some large cardinals like inaccessible and measurable cardinals. Martins Axiom and its consequences. Axiomatic development of set theory upto foundation axiom, Class and Class as models, relative consistency, absoluteness, Reflection principle, Mostowski collapse lemma, non-decidability of large cardinal axioms, Godel's second incompleteness theorem, Godel's constructible universe, Forcing lemma and independence of CH.

Suggested Texts:

1. S. M. Srivastava, A Course on Axiomatic Set Theory.
2. K. Kunen, Set theory. An introduction to independence proofs, Studies in Logic and the Foundations of Mathematics (102), North-Holland Publishing Co. (1980).
3. T. Jech, Set theory, Academic Press (1978).

Game Theory

A. Non-Cooperative Games: Games in normal form. Rationalizability and iterated deletion of never-best responses. Nash equilibrium: existence, properties and applications. Two-person Zero Sum Games. Games in extensive form: perfect recall and behavoiour strategies. Credibility and Subgame. Perfect Nash equilibrium. Bargaining. Repeated Games; Folk Theorems.

B. Introduction to Cooperative Games (TU Games).

Note: The course may be combined with the course "Game Theory-I" of M.Stat. 2nd year or the course "Game Theory-I" of the MSQE programme at ISI.

Automata, Languages and Computation

1. Automata and Languages: Finite automata, regular languages, regular expressions, equivalence of deterministic and non-deterministic finite automata, minimisation of finite automata, closure properties, Kleene's theorem, pumping lemma and its applications, Myhill-Nerode theorem and its uses. Context-free grammar, context-free languages, Chomsky normal form, closure properties, pumping lemma for CFL, pushdown automata.

2. Computability: Computable functions, primitive and partial recursive functions, universality and halting problem, recursive and recursively enumerable sets, parameter theorem, diagonalisation and reducibility, Rices theorem and its applications, Turing machines and its variants, equivalence of different models of computation and Church-Turing thesis.

3. Complexity: Time complexity of deterministic and non-deterministic Turing machines, P and NP, NP-completeness, Cook's theorem: other NP-complete problems.

Suggested Texts:

1. N. Cutland, Computability. An introduction to recursive function theory, Cambridge University Press (1980).

2. M. D. Davis, Ron Sigal and E. J. Weyuker, Computability, complexity, and languages. Fundamentals of theoretical computer science, Academic Press (1994).

3. J. E. Hopcroft and J. D. Ullman, Introduction to automata theory, languages, and computation, Addison-Wesley (1979).

4. H. R. Lewis and C. H. Papadimitriou, Elements of the theory of computation, Prentice-Hall (1981).

5. M. Sipser, Introduction to the theory of computation, PWS Pub Co, NY (1999).

6. M. R. Garey and D. S. Johnson, Computers and intractability. A guide to the theory of NP-completeness, W. H. Freeman and Co. (1979).

Note: The course may be combined with the corresponding course of M. Tech. 1st Year (ISI).

Advanced Fluid Dynamics

Inviscid incompressible fluid:

Two dimensional motion, stream function, complex potential and velocity, sources, sinks. Doublets and their images. Circle theorem, Blasiuss theorem, Kutta- Jokowaski theorem. Axisymmetric motion, Stokes stream function. Image of a source and a sink with respect to a sphere.

Vortex motion, vortex lines and filaments, systems of vortices, rectilinear vortices, vortex pair and doublets. A single infinite row of vortices, Karmans vortex sheet.

Linearised gravity waves, progressive waves in deep and shallow water, stationary waves, energy and group velocity, long waves and their energy, capillary waves.

Inviscid compressible fluid:

First and second law of thermodynamics, polytropic gas and its entropy, adiabatic and isentropic flow, propagation of small disturbances. Mach number, Mach cone, irrotional motion, Bernoullis Equation, pressure, density and temperature in terms of Mach number. Area velocity relations in one-dimensional flow, concept of subsonic and supersonic flows. Normal shock-wave, Rankine- Hugonoit and Prandtls relations in case of a plane shock wave.

Viscous incompressible fluid: Equations of motion of a viscous fluid, Reynold's number, circulation in a viscous liquid, Flow between parallel plates, flow through pipes of circular, elliptic and annular section under constant pressure gradient. Prandtl's concept of boundary layer.

Suggested Texts:

1. L. M. Milne-Thomson, Theoretical hydrodynamics, Macmillan (1960).

2. L. D. Landau and E. M. Lifshitz, Fluid mechanics, Course of Theoretical Physics, Vol. 6 Pergamon Press (1959).

3. H. Lamb, Hydrodynamics, Cambridge University Press (1993).

4. W. H. Besant and A. S. Ramsey, A treatise of Hydro-mechanics Part II, ELBS.

5. P. K. Kundu, Fluid mechanics, Academic Press.

Quantum Mechanics I

1.

(i) Physical Basis of Quantum Mechanics.

(ii) Old Quantum theory.

(iii) Uncertainty, Complimentarity and Duality.

(iv) Measurement problems.

(v) Heisenberg and Schrodinger representation.

2. (i) Schrodinger wave equation (ii) Perturbation theory.

3. Problem of two or more degrees of freedom without spherical symmetry; Stark effect.

4. Angular momentum, SU(2) algebra.

5. Three-dimensional Schrodinger equation. Problems with spherical symmetry. Harmonic Oscillator.

6. Scattering problem , differential cross section, phase shift, variational principle, SW transformation, Regge poles.

7. WKB approximation.

8. Particles with spin, Pauli matrices, Pauli-Schrodinger equation. Two and three body problems. Hydrogen atom in electric and magnetic field.

9. Quantum Statistics.

Suggested Texts:

1. L.I. Schiff, Quantum Mechanics.

2. J.J. Sakurai, Modern Quantum Mechanics.

3. L. D. Landau and E. M. Lifshitz, Quantum mechanics: non-relativistic theory, Course of Theoretical Physics Vol 3, Pergamon Press Ltd (1958).

4. L.M. Falicov, Group theory and its physical applications, Univ of Chicago Press (1966).

Quantum Mechanics II

Non stationary problems. Relativistic Dirac equation, Spinors. Scattering by a central force. Radiation theory. Quantization of Schrodinger field. Born approximation. Compton effect (

Klein Nishina formula). Bremsstrahlung. Symmetry and conservation laws. Quantum Probability and quantum Statistics. Supersymmetric Quantum Mechanics, SWKB. Path integral method.

Suggested Texts:

1. L.I. Schiff, Quantum Mechanics.
2. P.A.M. Dirac, The Principles of Quantum Mechanics, Oxford, Clarendon Press (1947).
3. P. A. M. Dirac, Spinors in Hilbert space, Plenum Press (1974).
4. M. E. Rose, Elementary theory of angular momentum, John Wiley.
5. R. P. Feynman and A. R. Hibbs, Quantum Mechanics and Path integrals.
6. L. D. Landau and E. M. Lifshitz, Statistical physics, Course of Theoretical Physics. Vol. 5. Pergamon Press Ltd (1958).
7. S. Flugge, Practical quantum mechanics, Springer-Verlag (1999).
8. H. Weyl, The theory of groups and Quantum Mechanics.

Analytical Mechanics

Generalised coordinates, Lagranges Equation. Examples of Lagranges equation. Conservation laws. Motion in a central field. Collision of particles. Small Oscillations. Rotating Coordinate systems. Inertial forces. Dynamics of a rigid body. Hamiltonian Mechanics.

Suggested Texts:

1. I. Arnold, Mathematical methods of classical mechanics GTM (60), Springer- Verlag (1978).
2. R. Abraham and J. E. Marsden, Foundations of mechanics Second edition, Benjamin/Cummings (1978).

Project I

Project II

Special Topics