# Indian Statistical Institute, Bangalore M.S. (QMS) First Year <br> Second Semester - Statistics for Decision Making II 

Final Exam Duration: 2 Hrs Date: Max Marks: 50

## This paper carries 60 Marks. Answer as many questions as you can.

1. Consider a simple linear regression model

$$
y_{i}=\beta_{0}+\beta_{1} x_{i}+\varepsilon_{i}
$$

Where $\beta_{0}$ and $\beta_{1}$ are the unknown parameter to be estimated from sample data. We assume that $\varepsilon_{i}$ are random variable with variance of $\sigma_{e}^{2}$. Prove the followings
a. $E\left(\epsilon_{i}\right)=0$
b. $\frac{S S_{E}}{n-2}$ is an unbiased estimator of $\sigma_{e}^{2}$ where $S S_{E}$ is the sum of square due to error (residuals)?
2. The performance of hand-held chain saws measured through deflection angle. The following table shows the deflection angles for six saws chosen at random from each of three different manufacturers. Is there any evidence that the manufacturers' products differ with respect to angle? Use $\alpha=0.01$.

| Manufacturer | Deflection Angle |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 32 | 17 | 24 | 39 | 28 | 43 |
| B | 45 | 50 | 44 | 33 | 46 | 61 |
| C | 57 | 49 | 48 | 41 | 52 | 54 |

Note: Deflection angle does not follow Normal Distribution
3. Regression methods were used to analyse the data from a study investigating the relationship between roadway surface temperature ( $x$ ) and pavement deflection (y). Summary quantities were $\mathrm{n}=20, \sum y_{i}=12.75, \sum y_{i}^{2}=$ 8.86, $\sum x_{i}=1478, \sum x_{i}^{2}=143215.8$, and $\sum x_{i} y_{i}=1083.67$
a. Calculate the least squares estimates of the slope and intercept. Graph the regression line.
b. Use the equation of the fitted line to predict what pavement deflection would be observed (with $95 \%$ CI or PI) when the surface temperature is 85.
c. What change in mean pavement deflection would be expected for a 1unit change in surface temperature?

$$
[3+4+3=10]
$$

4. An engineer who suspects that the surface finish $\left(\boldsymbol{y}_{\boldsymbol{i}}\right)$ of metal parts is influenced by the type of paint used and the drying time. He selected three drying times-20, 25, and 30 minutes - and used two types of paint. Three parts are tested with each combination of paint type and drying time. The data are as follows:

| Paint | Drying Time (min) |  |  |
| :---: | ---: | ---: | ---: |
|  | 20 | 25 | 30 |
| 1 | 74 | 73 | 78 |
|  | 64 | 61 | 85 |
|  | 50 | 44 | 92 |
| 2 | 92 | 98 | 66 |
|  | 86 | 73 | 45 |
|  | 68 | 88 | 85 |

Note: $n=18, \sum y_{i}=1322$ and $y_{i}^{2}=101598$
a. Write down the model.
b. State and test the appropriate hypotheses using the analysis of variance with $\alpha=0.05$.

$$
[2+8=10]
$$

5. To determine whether there really is a relationship between an employee's performance in the company's training program and his or her ultimate success in the job, the company takes a sample of 200 cases from its very extensive files and obtains the results shown in the following table:

| Job <br> Success | Training <br> Performance |  |
| :--- | ---: | ---: |
|  | Average | Excellent |
| Average | 54 | 44 |
| Excellent | 34 | 68 |

a. Use $\alpha=0.01$, to test whether performance in the training program and success in the job are dependent or not. In case it is dependent, explain the type of dependency.
6. Write short notes (any two) on the importance of the following in developing linear model.
a. Cross validation
b. Heteroscedasticity
c. Box-Cox transformation

