Indian Statistical Institute

SQC & OR Unit, Bangalore centre

MS(QMS)

SDM-2 : Mid Semester Exam 2020

This paper carries 60 Marks. Time: 2 Hrs Date: 3rd March,2020

Answer as many questions as you can, but the maximum score you can get is 50 only.

1. a) Let X be a random variable having probability mass function as

$$p(x) = p^x(1-p)^{1-x}, x = 0,1$$
$$= 0, \quad otherwise$$

Find the Maximum Likelihood Estimator (MLE) of p.

[10]

2. a) Let Z be a standard normal variable . Show that Z^2 follows a Chi-square distribution with 1 d.f.

b) Hence show that ${Z_1}^2+{Z_2}^2 + \cdots + {Z_k}^2$ is distributed as a Chi-square distribution with k d.f, where Z_i ' s (i=1,2,....,k) are independently distributed as Normal distribution with mean=0 and standard deviation =1.

[6+6=12]

3. A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimetres. Find a 99% confidence interval for the mean diameter of pieces from this machine, assuming an approximately normal distribution.

[8]

4. A soft-drink machine at a cinema hall is regulated so that the amount of drink dispensed is approximately normally distributed with a mean of 200 millilitres and a standard deviation of 15 millilitres. The machine is checked periodically by taking a sample of 9 drinks and computing the average content. If x bar falls in the interval $191 < x\ bar < 209$, the machine is thought to be operating satisfactorily; otherwise, we conclude that $\mu = 200$ millilitres.
   (a) Find the probability of committing a type I error when $\mu = 200$ millilitres.
   (b) Find the probability of committing a type II error when $\mu = 213$ millilitres.

[7+8=15]

5. Explain the following with example:
   a) Efficiency of an estimator
   b) Prediction interval of a future observation
   c) Type-I & Type-II error

[3x5=15]