Indian Statistical Institute

Bangalore centre

Mid Semester Exam 2019   SDM-2

This paper carries _60 Marks_.

Answer as many questions as you can, but the maximum score you can get is 50 only.

Date: 01-03-2019   **Max.marks:50**   **Time: 3 Hrs**

1. From a normal population of measurements with unknown mean μ and unknown variance σ^2, a random sample of size n is drawn.
Derive a $100(1 - \alpha)$% prediction interval of a the next observation x_0.

[7]

2. Let X be a random variable with mean μ and variance σ^2 and let $X_1, X_{2,......}, X_n$ be a random sample of size n from the population of X . Show that the estimate
$G = K \sum_{i=1}^{n-1}(X_{i+1} - X_i)^2$ is unbiased for estimating σ^2 for an appropriate choice of K.
Find the appropriate value of K.

[6]

3. Let X_1 and X_2 be independent chi-square random variables with n_1 and n_2 degrees of freedom respectively. Then show that the ratio $F = \dfrac{X_1/n_1}{X_2/n_2}$ has the probability density function

$g(f) = \{\Gamma^{(n_1 + n_2)/2} (n_1/ n_2)^{n_1/2} f^{(\frac{n_1}{2} - 1)}\}/\Gamma^{(n_1/2)} \Gamma^{(n_2/2)} [(n_1 f/ n_2) + 1)]^{(n_1 + n_2)/2}$, $0 < f < \infty$.

[10]

4. A soft-drink machine at a steak house is regulated so that the amount of drink dispensed is approximately normally distributed with a mean of 200 millilitres and a standard deviation of 15 millilitres. The machine is checked periodically by taking a sample of 9 drinks and computing the average content. If x-bar falls in the interval $191 <$ x-bar < 209, the machine is thought to be operating satisfactorily; otherwise, we conclude that $\mu \neq 200$ milliliters.
 (a) Find the probability of committing a type I error when $\mu = 200$ millilitres.
 (b) Find the probability of committing a type II error when $\mu = 215$ millilitres.

[6+6=12]

5. In a random sample of $n = 500$ families owning television sets in the city of Bangalore,
 it is found that $x = 340$ subscribe to a particular movie channel MOVIE. Find a 95% confidence interval for the true proportion of families with television sets in this city that subscribe to MOVIE. How large a sample is required if we want to be 95% confident that our estimate of the said true proportion is within 0.02 of the true value?

[6+4=10]

6. Explain the following with example:
   a) Efficiency
   b) Method of Maximum Likelihood Estimation
   c) Type-I & Type-II error

[3x5=15]