

Indian Statistical Institute, Bangalore
M.S. (QMS) First Year
First Semester – Statistics for Decision Making II

Mid Term Exam Duration: 2 Hrs Date: February 21, 2017 Max Marks: 50

This paper carries 60 Marks. Answer as many questions as you can.

1. Let U_1 and U_2 be independent random variables. Suppose that U_1 is chi-square with ν_1 degrees of freedom while $U = U_1 + U_2$ is chi-square with ν degrees of freedom, where $\nu > \nu_1$. Then prove that U_2 is chi-square random variable with $\nu - \nu_1$ degrees of freedom.

(7)

2. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be two unbiased estimators of θ . Show that $\hat{\theta}_3 = a\hat{\theta}_1 + (1-a)\hat{\theta}_2$, ($0 \leq a \leq 1$), is also an unbiased estimator of θ . How should you choose the value of a so as to minimize the variance of $\hat{\theta}_3$?

(7)

3. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from $N(0, \sigma^2)$. Show that sample variance $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$ is a consistent estimator of σ^2 .

(9)

4. (a) An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

(b) How large a sample is needed, if we wish to be 99% confident that our sample mean will be within 10 hours of the true mean? (5+5=10)

5. The sodium content of thirty 300-gram boxes of organic corn flakes was determined. The data (in milligrams) are as follows:

131.15, 130.69, 130.91, 129.54, 129.64, 128.77, 130.72,
128.33, 128.24, 129.65, 130.14, 129.29, 128.71, 129.00, 129.39,
130.42, 129.53, 130.12, 129.78, 130.92, 131.15, 130.69, 130.91,
129.54, 129.64, 128.77, 130.72, 128.33, 128.24, and 129.65.

Can you support a claim that mean sodium content of this brand of cornflakes is 130 milligrams? Use $\alpha = 0.05$. (7)

6. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 Volts, and the manufacturer wishes to test $H_0: \mu = 5$ Volts against $H_1: \mu \neq 5$ Volts, using $n = 8$ units.

- (a) The acceptance region is $4.85 \leq \bar{x} \leq 5.15$, Find the value of $\alpha =$ level of significance.
(b) Find the power of the test when the true mean output voltage is 5.1 Volts.

(5+5=10)

7. Explain any **TWO** the following with example:

- a) Efficiency
- b) Method of Moments in Estimation
- c) Type-I & Type-II error

(2x5=10)