Indian Statistical Institute, Bangalore

M.S. (QMS) First Year

Second Semester – Statistics for Decision Making II

Mid Term Exam   Duration: 2 Hrs   Date: February 24, 2016   Max Marks: 50

This paper carries 60 Marks. Answer as many questions as you can.

1. From a normal population of measurements with unknown mean μ and known variance σ^2, a random sample of size n is drawn.
Derive a $100(1 - \alpha)\%$ prediction interval of a future observation x_0. (7)

2. Let us define $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$, show that $E(s^2) = \frac{n-1}{n}\sigma^2$, where σ^2 is the population variance. (8)

3. It is known that a sample consisting of the values 1n2, 11.2, 13.5, 12.3, 13.8, and 11.9 comes from a population with the density function

$f(x; \vartheta) = \frac{\theta}{x^{\theta+1}}$

0, elsewhere,

where $\vartheta > 0$. Find the maximum likelihood estimate of ϑ. (7)

4. A manufacturer of car batteries claims that the batteries will last, on average, 3 years with a variance of 1 year. If 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, construct a 95% confidence interval for σ^2 and decide if the manufacturer's claim that $\sigma^2 = 1$ is valid. Assume the population of battery lives to be approximately normally distributed. (6)

5. Test the hypothesis that the average content of containers of a particular lubricant is 10 litres if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 litres.
Use a 0.01 level of significance and assume that the distribution of contents is normal. (6)

6. A soft-drink dispensing machine is said to be out of control if the variance of the contents exceeds 1.15 decilitres. If a random sample of 25 drinks from this machine has a variance of 2.03 decilitres, does this indicate at the 0.05 level of significance that the machine is out of control? Assume that the contents are approximately normally distributed. (6)

7. Explain the following with example: (4x5=20)
    a) Unbiassedness
    b) Efficiency
    c) Method of Moments in Estimation
    d) Type-I & Type-II error