Indian Statistical Institute, Bangalore

MS(QMS)

Second Semester - Statistics for Decision Making II

Mid-Semester Exam Duration: 2 hours Date : March 05, 2015

This paper has questions of 65 marks. Answer as many questions as you can, but the maximum you can score is 50.

1. Let $X_1, X_2, ..., X_n$ be a random sample from a population following Gamma Distribution with parameters α and β. Show that the Maximum Likelihood Estimate (MLE) of $\mu = \alpha\beta$ is the sample mean $\widehat{\mu} = \overline{x}$. [10]

2. Let $X_1, X_2, ..., X_{n1}$ be a random sample from the Normal Population $N(\mu_1, \sigma^2)$ and $Y_1, Y_2, ..., Y_{n2}$ be and independent random sample from the Normal Population $N(\mu_2, \sigma^2)$, where σ^2 is assumed to be known. Derive a $100(1-\alpha)\%$ confidence interval for $(\mu_1 - \mu_2)$ and interpret its meaning. [8+2 = 10]

3. If $Z_1, Z_2, ..., Z_K$ are K independent Standard Normal Variables, then show that $Y = \sum_{i=1}^{k} Z_i^2$ follows a Chi-square distribution with K degrees of freedom. [10]

4. A manufacturer of Car batteries claims that the batteries will last, on an average 3 years with a variance of 1 year. If 5 of the batteries have lifetimes of $1.9, 2.4, 3.0, 3.5$ and 4.2 years, then construct a 95% confidence interval of σ^2 and decide if the manufacturer's claim that $\sigma^2 = 1$ is valid. Assume the population of battery lives to be normally distributed. [10]

5. Following data show the number of work hours lost per day on a construction project due to weather related incidents for 11 work days. The number of hours lost per day for the same are: $8.8, 12.5, 5.4, 12.8, 9.1, 14.7, 8.8, 12.2, 13.3, 6.9, 2.2$. Assuming lost work hours are normally distributed, is there evidence to conclude that the mean number of work hours lost per day is greater than 8 hours? [10]

6. A melting point test of $n = 10$ samples of a binder used in manufacturing a rocket propellant resulted in $\overline{X} = 154.2^0F$ Assume that melting point is normally distributed with $\sigma = 1.5^0F$.

   (a) Test $H_0 : (\mu = 155) Versur H_1 : (\mu \neq 155)$ using $\alpha = 0.01$

   (b) What is the P - value for this test?

   (c) What is the value of β when the true mean is $\mu = 150$?

   (d) What value of n would be required if we want $\beta < 0.1$ when $\mu = 150$? Assume $\alpha = 0.01$ [3+4+4+4 = 15]