

Statistics for Decision Making - I

Full Marks: 40 Time : 2 hr 30 minutes

Answer 1 and 2 and any two from the rest.

1. Observations are taken on yield of hay (x_1) in 100 cuts/ acre, spring rainfall (x_2) in inches and accumulated spring temperature (x_3) in Fahrenheit for 20 years. The following estimates are obtained.

Sample mean vector: $\bar{x} = (28.02, 4.91, 59.00)'$,

Vector of sample standard deviations: $s = (4.42, 1.10, 85.00)'$

and the correlation matrix is

$$\begin{pmatrix} 1 & 0.80 & -0.40 \\ & 1 & -0.56 \\ & & 1 \end{pmatrix}$$

Find

- The multiple linear regression equation of x_1 on x_2 and x_3 .
- The multiple correlation coefficient of x_1 with x_2 and x_3 . And hence make a performance analysis of the regression equation
- The partial correlation coefficient between x_1 and x_3 eliminating the effect of x_2 .

(4+3+3)

2. a) In an examination, 9 students obtained the following marks in Economics and Statistics. Find Spearman's rank correlation coefficient. (6)

Roll No.	1	2	3	4	5	6	7	8	9
Economics	45	60	32	45	32	32	58	56	47
Statistics	51	51	38	54	54	38	62	58	38

- b) If the pdf of X is given by

$$f(x) = \begin{cases} 2xe^{-x^2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Then determine the pdf of $Y = X^2$ (4)

- Distinguish between correlation and association. (3)
 - Explain sensitivity and specificity in the context of 2 X 2 contingency table. (3)
 - Given $n = 2500$, $f_A = 420$, $f_{AB} = 85$ and $f_B = 670$, prepare a (2X2) contingency table and compute Yule's coefficient of association. (4)
- Define pseudo random number. (2)
 - Draw a random sample of size 10 from a 2 parameter Weibull distribution with pdf

$$f(x) = \frac{\beta}{\alpha^\beta} x^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^\beta}, \quad x \geq 0$$

Consider $\alpha = 2$ and $\beta = 0.7$ (5)

c) Derive the expression for the mgf of chi-square distribution. (3)

5. a) Fit a linear regression equation to the following data and compute the coefficient of determination. (5+2)

Age in years	50	42	72	36	68	47	55	49
Blood pressure	147	125	160	118	149	128	150	145

- b) Find \bar{x} and \bar{y} from the regression equations given by $y = 0.7 + 1.1x$ and $x = 0.25 + 0.687y$ (3)

6. a) Derive the expression for $\text{Var}(\bar{y})$ in the context of SRSWOR. (5)

b) Discuss in brief, different types of sample allocation schemes in the context of stratified random sampling. (5)