Indian Statistical Institute, Bangalore

M.S. (QMS) First Year

First Semester – Reliability Maintainability and Safety I

Max. Marks: 50 Duration: 3 Hrs Date: September 14, 2023

Answer any 5 from the following

1. a) Define sample space. Give examples of i) finite sample space, ii) countably infinite sample space, iii) infinite/ uncountable sample space. (5)
   b) Suppose, a sample space S consists of 4 elements a_1, a_2, a_3 and a_4. Which function defines a probability space on S? Give justification for each of the cases
   i) $P(a_1) = ½$, $P(A_2) = 1/3$, $P(A_3) = 2/3$, $P(a_4) = 1/5$
   ii) $P(a_1) = ½$, $P(A_2) = 1/4$, $P(A_3) = -1/4$, $P(a_4) = ½$
   iii) $P(a_1) = ½$, $P(A_2) = 1/4$, $P(A_3) = 1/8$, $P(a_4) = 1/8$

2. a) State and prove Baye's Theorem. (6)
   b) There are three machines in a factory labelled as A, B and C. They produce respectively 50%, 30% and 20% of the total number of items of the factory. Also, percentages of defective items produced by them are 3, 4, 5 respectively. If an item is selected at random, what is the probability that, the item will be defective? (4)

3. a) If $X_1, X_2, \ldots, X_n$ are i.i.d., then show that the mgf of the sum of the random variables is same as the n^{th} power of their common mgf. (5)
   b) Find the cdf for the following pdf. (5)

$$f(x) = \begin{cases} x, & for\ 0 \le x \le 1 \\ 2 - x, & for\ 1 < x \le 2 \\ 0, & Otherwise \end{cases}$$

4. a) Let

$$f(x) = \begin{cases} \frac{x}{n(n+1)}, & for\ x = 1,2,\ldots,n \\ 0, & otherwise \end{cases}$$

   Check whether f(x) is a pmf. If not, make it a pmf. (5)
   b) Find the expectation and Variance of X. (5)

5. a) Team A has probability 2/3 of winning a game, whenever it plays. If the team plays 4 games, find the probability that A wins i) exactly 2 games, ii) atleast 1 game, iii) more than half of the games. (5)
   b) Derive the expression for the variance of negative binomial distribution. (5)
6. a) Discuss the lack of memory property of exponential distribution. (5)

   b) Family income of a country is distributed as N($25000, $$10000^2$). If the poverty level is $10,000, what percentage of the population lives below poverty level? (given that $\Phi(1.5) = 0.9332$) (5)

7. a) Find the expression for the median of Normal (μ, σ^2) distribution. (4)
   b) Derive the expression for the mgf of binomial distribution. Hence show that, if X_1~Bin(n_1, p) and X_2~Bin (n_2, p), then $X_1 + X_2$~Bin ($n_1 + n_2$, p) (6)