

Indian Statistical Institute, Bangalore

M.S. (QMS) First Year

First Semester - Reliability, Maintainability and Safety-I

Final Exam

Time: 3 hours

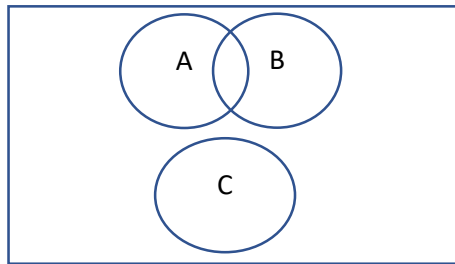
Date: 27/12/2021

Maximum Marks: 50

This paper has two parts. Answer as many questions as you can. You can score maximum of 30 for Part - A and 20 for Part - B.

Part – A: Probability

1. Three events A, B and C are shown in Venn diagram in the figure. Shade the region that corresponds to each of the following events.



a) $(B \cup C)^c$ b) $(A \cap B) \cup C$ c) $(A \cap B)^c \cup C$ d) $A - B$

[1.5+1.5+1.5+1.5 = 6]

2. Each year ratings are compiled concerning the performance of new cars during the first 90 days of use. Suppose the car have been categorized according to whether a car needs warranty related repair (yes or no) and the country in which the company manufacturing a car is based (within India or outside). Based on the data collected, the probability that the new car needs a warranty repair is 0.04, the probability that the car was manufactured by an India-based company is 0.60, and the probability that the new car need a warranty repair and was manufactured by an India-based company is 0.025. What is the probability that a new car selected at random,
- Needs a warranty repair or was manufactured by an India-based company?
 - Needs a warranty repair or was not manufactured by an India-based company?
- [3+3 = 6]
3. a. If $P(A/B) = 0.3$, $P(B) = 0.8$ and $P(A) = 0.3$, are the event B and complement of A independent?
b. If $P(A) = 0.2$, $P(B) = 0.2$ and A and B are mutually exclusive, are they independent?
- [3+3 = 6]
4. A construction company is determining whether it should submit a bid for a new shopping center. In the past, company's competitor has submitted bids 70% of the time. If the competitor company does not bid on a job, the probability that the company will get the job

is 0.50. If the competitor bid on a job, the probability that the construction company will bet the job is 0.25.

- a) If the construction company gets the job, what is the probability that the competitor company did not bid?
- b) What is the probability that the construction company will get the job?

[3+3 = 6]

5. According to the Govt. report, 35% of new business in 2015 were started by women and 65% of new business were started by men. Twenty percent of 2015 new business by women had revenues of Rs. 50,000 crore and above, whereas 24 % of new business by men had revenues of Rs.50,000 crore and above.

- a) If a new business started in 2015 is selected at random and the business had revenue of Rs.50,000 crore and above, what is the probability that the business started by a woman?
- b) If a new business started in 2015 is selected at random, what is the probability that the new business had a revenue of Rs. 50,000 crore and above?

[3+3 = 6]

6. Heart failure is due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances or foreign objects. Natural occurrences are caused by arterial blockage, disease, and infection. Assume that causes of heart failure between individuals are independent.

- (a) What is the probability that the first patient with heart failure that enters the emergency room has the condition due to outside factors?
- (b) What is the probability that the third patient with heart failure that enters the emergency room is the first one due to outside factors?

[3+3 = 6]

Part – B: Reliability, Maintainability and Safety

7. The CDF for time to failure for a system is given by

$$F(t) = 1 - \frac{100}{(t + 10)^2} ; t \geq 0$$

- a) Find the i) Reliability function and ii) Failure rate Function (Hazard rate).
- b) Check if it is IFR (increasing failure rate), DFR (decreasing failure rate), or CFR (constant failure rate).

[5 + 5 = 10]

8. Suppose lifetime of a product T follows Gamma distribution with the following pdf:

$$f(t) = \frac{\lambda^\alpha t^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda t}, t \geq 0, \alpha > 0, \lambda > 0.$$

Show that the lifetime of the product is DFR for $0 < \alpha < 1$ and IFR for $\alpha > 1$.

What is the name of the distribution if $\alpha = 1$.

[4 + 4 + 2 = 10]

9. Write a short note on Bathtub Curve.

[5]