Indian Statistical Institute, Bangalore

M S(QMS) First Year

First Semester - Probability

Mid-Semester Exam | Duration: 2hrs | Date: Sept 12, 2014

Answer questions 1, 2, 3 and one from 4 and 5

1. A balanced die is rolled n times independently where $n \geq 2$. Let X denote the number of times 6 dots show up and Y denote the number of times 5 dots show up in these n rolls.
(a) What is the joint probability distribution of (X, Y)?
(b) Find the probability distribution of $Z = X + Y$.
(c) Find $E(Z)$, $Var(Z)$ and $Cov(X, Z)$. [12]

2. Suppose the joint probability mass function of (X, Y) is given by

$$f_{X,Y}(x, y) = \begin{cases} p^2(1-p)^y & \text{if } 0 \leq x \leq y < \infty,\ x \text{ and } y \text{ are integers;} \\ 0 & \text{otherwise,} \end{cases}$$

for $0 < p < 1$.
(a) Find the marginal probability mass functions of X and Y.
(b) Are X and Y independent?
(c) What is the name of the probability distribution of Y? [12]

3. Assume that there are equal number of males and females in a particular population. Suppose that 5% of men and 1% of women are colour-blind. A colour-blind person is chosen at random. What is the probability of this person being male? [8]

4. For events A, B and C defined on the same probability space, show that
(a) $P(A \cap B) \geq P(A) + P(B) - 1$, and
(b) $P(A \cap B \cap C) \geq P(A) + P(B) + P(C) - 2$. [8]

5. Suppose $X \sim$ Bernoulli(p), $Y \sim$ Poisson(λ) and these two are independently distributed. Let $Z = X + Y$. Find
(a) the p.m.f of Z;
(b) $E(Z)$ and $Var(Z)$. [8]