Indian Statistical Institute, Bangalore
MS (QMS) First Year
Second Semester - Operations Research II

Final Exam — Date: April 29, 2019
Maximum marks: 100 — Duration: 3 hours

This question paper carries 105 marks. You may answer as many questions as you can, but your maximum score will be limited to 100.

1. (a) Develop the procedure for generating (simulating) Poisson Distributed random variable from Uniform Random Numbers, using Inverse Transformation method.

   (b) Generate a Poisson Distributed Random Variable for a Poisson Distribution with mean $\lambda = 3$ events/hour, during time $t = 1.6$ hours. Use $U(0,1)$ random numbers to solve this.

   (c) The Random Variable X has probability density function:

   $$f(x) = Ce^{x} \qquad 0 < x < 1$$

   Find the value of constant C and then give a method for simulating such a random variable. Generate 2 samples for this Random variable X, using Uniform Distribution Random Numbers.

   (d) A factory manufactures some electronic device with a production defective rate 14%. These devices are packed in small packets whose capacity is 7 devices/packet & sent to final inspection department. Use simulation technique to calculate the probability that a randomly selected packet will have maximum 1 defective device. Compare your result with the theoretical value. Use Uniform Distribution Random Numbers to solve this question. *Your simulation experiment sample size should not be less than 20.*

   [10 + 6 + 8 + 16 = 40]

2. (a) For the Queuing System: $(M/M/c)(GD/N/\infty)$, $c \leq N$, Derive the formula for:

   (i) $p_0(t)$ = probability of no customer in the system at time t
   (ii) $p_n(t)$ = probability of n customers in the system at time t
   (iii) L_q = Expected length of the Queue

   You may use the result of generalized steady state model for p_n as a starting point.

   (b). For the Queuing system: $(M/M/1)(GD/\infty/\infty)$, show that

   (i) The expected number in the queue, given that the queue is not empty is $= [1/(1-\rho)]$, where $\rho = \lambda/\mu$

   (ii) The average duration of busy period $= [1/(\mu - \lambda)]$

   Where all the symbols have their usual meanings.

(c) In a car wash service facility, information gathered indicates that cars arrive for service according to a Poisson distribution with mean 5 cars per hour. The time for washing and cleaning each car varies but is found to follow an exponential distribution with mean 10 minutes per car. Answer the following: (*you may make any realistic assumption that may be required.*)

(i) What is expected queue length L_q?
(ii) What should be the minimum size of the parking space such that any arriving car will be able to park at least 80% of the time?
(iii) Percentage of time facility is idle?
(iv) Suppose now that a parking facility for 5 cars parking is build and whenever it is full other cars go elsewhere for service. Calculate how many cars the facility will lose per day of 8 hours on an average, due to limited parking space. Also calculate the expected number of parking spaces occupied.

[10 + 10 + 20 = 40]

3. A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending node numbers.

Activity (i – j)	Estimated duration (in weeks) Optimistic	Most likely	Pessimistic
1 – 2	1	1	7
1 – 3	1	4	7
1 – 4	2	2	8
2 – 5	1	1	1
3 – 5	2	5	14
4 – 6	2	5	8
5 – 6	3	6	15

(i) Draw the project network
(ii) Find the expected duration and variance for each activity
(iii) What is the expected project length?
(iv) Calculate the variance and standard deviation of the project length.
(v) What is the probability that the project will be completed at least 2 weeks earlier than the expected completion time?
(vi) If the deadline is 20 weeks, what is the probability that the project will fail to meet the deadline?

[25]
