Indian Statistical Institute, Bangalore

MS (QMS)

First Year, Second Semester

Operations Research II

Mid-term Examination — Date : Feb. 26, 2018

Maximum marks: 50 — Time: 3 hours

Answer all the questions

(1) Develop Branch and Bound tree for each of the following problem. For convenience, always select x_1 as the branching variable at node 0.

$$\begin{aligned} \text{Maximize } z &= 2x_1 + 3x_2 \\ \text{Subject to } 5x_1 + 7x_2 &\leq 35 \\ 4x_1 + 9x_2 &\leq 36 \\ x_1, x_2 &\geq 0 \text{and integer} \end{aligned}$$

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(2) Use cutting plane method to solve the following, Integer programming problem.

$$\begin{aligned} \text{Maximize } z &= 7x_1 + 10x_2 \\ \text{Subject to } -x_1 + 3x_2 &\leq 6 \\ 7x_1 + x_2 &\leq 35 \\ x_1, x_2 &\geq 0 \text{and integer} \end{aligned}$$

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(3) A farmer in the Midwest has 1,000 acres of land on which she intends to plant corn, wheat, and soybeans. Each acre of corn costs \$100 for preparation, requires 7 workers-days of labor, and yields a profit of \$30. An acre of wheat costs \$120 to prepare, requires 10 worker-days of labor, and yields a profit \$40 profit. An acre of soybeans costs \$120 to prepare, requires 8 worker-days, and yields \$20 profit. The farmer has taken out a loan of \$80,000 for crop preparation and has contracted with a union for 6,000 worker-days of labor. A midwestern granary has agreed to purchase 200 acres of corn, 500 acres of wheat, and 300 acres of soybeans. The farmer has established the following goals, in order to their importance:

(a) To maintain good relations with the union, the labor contract must be honored; that is, the full 6,000 worker-days of labor contracted for must be used.
(b) Preparation costs should not exceed the loan amount so that additional loans will not have to be secured.
(c) The farmer desires a profit of at least \$ 105,000 to remain in good financial condition.
(d) Contracting for excess labor should be avoided.
(e) The farmer would like to use as much of the available acreage as possible.

(f) The farmer would like to meet the sales agreement with the granary. However, the goal should be weighted according to profit returned by each crop.

Formulate a goal programming model to determine the number of acres of each crop the farmer should plant to satisfy the goals in the best possible way.

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(4) Megan Moppett is a sales representative for Technical Software Systems (TSS), and she receives a commission for every new system installation she sells to a client. Her earnings during the past few years have been very high, and she wants to invest in a mutual fund. She is considering three funds: the Temple Global Fund, the Alliance Blue Chip, and the Madison Bond Fund. She has three criteria for selection-potential return (based on historical trends and forecasts),risk, and the fund's load factor. Megan's pairwise comparisons for the funds for each of their criteria and her pairwise comparison of the three criteria are as follows:

Return			
Fund	Global	Blue Chip	Bond
Global	1	1/4	2
Blue Chip	1	1	6
Bond	1/2	1/6	1

Risk			
Fund	Global	Blue Chip	Bond
Global	1	2	1/3
Blue Chip	1/2	1	1/5
Bond	3	5	1

Load			
Fund	Global	Blue Chip	Bond
Global	1	1	1/3
Blue Chip	1	1	1/3
Bond	3	3	1

Criterion	Return	Risk	Load
Return	1	3	5
Risk	1/3	1	2
Load	1/5	1/2	1

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(5) (a) In a steady state queueing system with mean arrival rate λ and arrival distribution *completely random*, prove that the number of arrivals in a given interval of time T follows a Poisson distribution with averageλT.

(b) For a $(M/M/1/) : (GD/\infty/\infty)$ Queueing system in steady state, with $p_n = p^n p_o$, where symbols have their usual meanings, derive:

(i) Expected number in the system.
(ii) Expected number in the queue.
(iii) Expected length of the non-empty system.

(c) A fast food restaurant has one drive-in window. Cars arrive according to a Poisson distribution at the rate of 2 cars every 5 minutes. The space in front of the window can accommodate at most 10 cars, including the one being served. Other cars can wait outside the space, if necessary. The service time per customer is exponential, with a mean of 1.5 minutes. Determine: $[6 + 6 + 6 = 18]$

(i) The probability that the facility is busy.
(ii) The expected number of customers waiting to be served.
(iii) The probability that the waiting line will exceed the 10-space capacity.