# Indian Statistical Institute, Bangalore <br> M.S. (QMS) First Year <br> First Semester - Operations Research II 

Mid Term Exam Duration: 2 Hrs Date: February 25, 2017 Max Marks: 50
Questions 1-3 carry a total of 40 marks and question 4 caries 10 marks.

1. Jobco is planning to produce at least 2000 widgets on three machines. The minimum lot size on any machine is 500 widgets. The following table gives the pertinent data of the situation:

| Machine | Cost | Production Cost / Unit | Capacity |
| :---: | :---: | :---: | :---: |
| 1 | 300 | 2 | 600 |
| 2 | 100 | 10 | 800 |
| 3 | 200 | 5 | 1200 |

Formulate the problem as an Integer Linear Program.
(Hint: Let $X_{j}=$ number of widgets produced on machine $j$.
$j=1,2,3$
$y_{j}=1$ if machine $j$ is used and 0 if machine $j$ is not used).

Write down the appropriate objective function and the constraints.
2. Use Branch and Bound algorithm to solve the following problem.

Maximize $Z=18 X_{1}+14 X_{2}+8 X_{3}+4 X_{4}$
Subject to $15 X_{1}+12 X_{2}+7 X_{3}+4 X_{4}+X_{5} \leq 37$
$X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$ are $0-1$ variables,
(Hint: Write down the LP relaxation: Start with initial solution $X_{1}=X_{2}=X_{3}=1, X_{4}=0.75$, $X_{5}=0$ and use B-B method).
3. Use Cutting Plane algorithm to solve.

Maximize $Z=3 X_{1}+X_{2}+3 X_{3}$
Subject to
$-\mathrm{X}_{1}+2 \mathrm{X}_{2}+\mathrm{X}_{3} \leq 4$
$4 X_{2}+3 X_{3} \leq 2$
$X_{1}-3 X_{2}+2 X_{3} \leq 3$
$X_{1}, X_{2}, X_{3}, \geq 0$ and integer,

Where the optimal tableau for the above LP is given by

| Basic | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{X}_{6}$ | Solution |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Z | 0 | 0 | 0 | 2 | 3 | 5 | 29 |
| $\mathrm{X}_{3}$ | 0 | 0 | 1 | $4 / 9$ | $1 / 9$ | $4 / 9$ | $10 / 3$ |
| $\mathrm{X}_{2}$ | 0 | 1 | 0 | $1 / 3$ | $1 / 3$ | $1 / 3$ | 3 |
| $\mathrm{X}_{1}$ | 1 | 0 | 0 | $1 / 9$ | $7 / 9$ | $10 / 9$ | $16 / 3$ |

4. Suppose that all car owners fill up when their tanks are exactly half full.At the presenttime, an average of 7.5 customers per hour arrive at a single-pump gas station. It takes an average of 4 minutes to service a car. Assume that arrival is Poisson and service times is exponential.
a) For the present situation, compute $L$ and $W$.
b) Suppose that a gas shortage occurs and panic buying takes place. To model the phenomenon, suppose that all car owners now purchase gas when their tank are exactly three-fourths full. Since each car owner is now putting less gas into the tank during each visit to the station, we assume that the average service time has been reduced to $31 / 3$ minutes. How has panic buying affected $L$ and $W$ ?
