INDIAN STATISTICAL INSTITUTE
Operations Research - II

Time:2:30 hrs **Total Marks: 50**

Answer any five questions [Neat presentation is necessary]

1. Consider the problem

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{Subject to}$$

$$x_1 \leq 2$$

$$x_2 \leq 2$$

$$x_1 + x_2 \geq 7/2$$

$$x_1, x_2 \geq 0 \text{ and integers}$$

   Find the optimal solution by Branch and Bound Techniques. Draw the diagram solution inside nodes. (10marks)

2. Machinists who work at a tool-and-die plant must check out tools from a tool center. An average of ten machinists per hour arrive seeking tools. At present, the tool center is staffed by a clerk who is paid \$6 per hour and who takes an average of 5 minutes to handle each request for tools. Since each machinist produces \$10 worth of goods per hour, each hour that a machinist spends at the tool center costs the company \$10. The company is deciding whether or not it is worthwhile to hire (at \$4 per hour) a helper for the clerk. If the helper is hired, the clerk will take an average of only 4 minutes to process requests for tools. Assume that service and interarrival times are exponential. Should the helper be hired? (10 marks)

3. Gandhi Cloth Company is capable of manufacturing three types of clothing: shirts, shorts, and pants. The manufacture of each type of clothing requires that Gandhi have the appropriate type of machinery available. The machinery needed to manufacture each type of clothing must be rented at the following rates: shirt machinery, \$200 per week; shorts machinery, \$150 per week; pants machinery, \$100 per week. The manufacture of each type of clothing also requires the amounts of cloth and labor shown in Table 1. Each week, 150 hours of labor and 160 sq yd of cloth are available. The variable unit cost and selling price for each type of clothing are shown in Table 2. Formulate an IP whose solution will maximize Gandhi's weekly profits. (10 marks)

   Table 1:

Clothing Type	Labor(Hours)	Cloth(Square Vards)
Shirt	3	4
Shorts	2	3
Pants	6	4

Table 2:

Clothing Type	Sales Price($)	Variable Cost($)
Shirt	12	6
Shorts	8	4
Pants	15	8

4. For the $(M|M|I) - (\infty|FIFO)$ Model derive expressions for the following: Assume Poisson arrivals with average λ and exponential service time with parameter μ

   (a) Probability of Queue size is greater than or equal to n, the number of customers.

   (b) Average number of customers in the system.

   (c) Average length of non empty Queue.

   (d) Average waiting time of a customer in the Queue.

   (e) Average waiting time that a Customer waits in the system.
   (three marks for each part)

5. Suppose that the demand for a product is 30 units per month and the items are withdrawn at a constant rate. The setup cost each time a production run is undertaken to replenish inventory is $ 15. The production cost is $1 per item, and the inventory holding cost is $0.30 per item per month.

   (a) Assuming shortages are not allowed, determine how often to make a production run and what size it should be.

   (b) If shortages are allowed but cost $3 per item per month, determine how often to make a production run and what size it should be. (10 marks)

6. A television manufacturing company produces its own speakers, which are used in the production of its television sets. The television sets are assembled on a continuous production line at a rate of 8,000 per month, with one speaker needed per set. The speakers are produced in batches because they do not warrant setting up a continuous production line, and relatively large quantities can be produced in a short time. Therefore, the speakers are placed into inventory until they are needed for assembly into television sets on the production line.

   The company is interested in determining when to produce a batch of speakers and how many speakers to produce in each batch. Several costs must be considered:
   1. Each time a batch is produced, a setup cost of $12,000 is incurred.

   2. The unit production cost of a single speaker (excluding the setup cost) is $11, independent of the batch size produced.

   The production of speakers in large batches leads to a large inventory. The estimated holding cost of keeping a speaker in stock is $0.30 per month.
   a). Find the EOQ and the Total production cost. (3marks)
   Suppose now that the unit cost for every speaker is c_1 = $11 if less than 10,000 speakers are produced, c_2 =$10 if production falls between 10,000 and 80,000 speakers, and c_3 =$9 if production exceeds 80,000 speakers.
   b).What is the optimal policy? (7 marks)