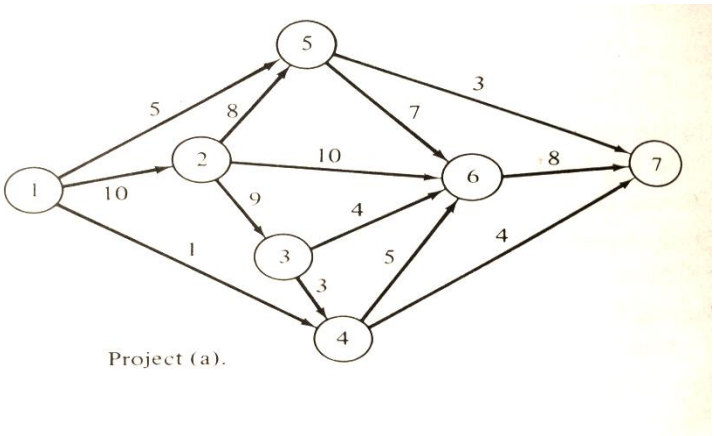
Indian Statistical Institute, Bangalore

M.S. (QMS) First Year
Second Semester – Operations Research II
FINAL EXAMINATION

Date: May 02, 2015 Duration: 3 Hours Maximum Marks: 100

1. (a) A game board has 3x3 equal squares. You are required to fill each square with a number between 1 to 9 [each number can be used only once] such that the sum of numbers in each row, each column, and each diagonal equals 15. Formulate this Problem as an Integer Linear Program (ILP).

   (b) Consider the following PERT project (network). The values given there are the time required to complete the concerned activities.



Project (a).

   (i) Compute the Total Float (TF) & Free Float (FF) for each activity and find the Critical Path.
   (ii) Suppose now that above activity times are probabilistic, following normal distribution, with mean times same as given in the diagram and Standard Deviations as shown in the following table. What is the probability that the Project will be completed in 38 days or less? Also calculate the probability that event (6) will be completed in 30 days or less and event (3) will require more than 20 days?

[6+8+8]

Activity	S.D.	Activity	S.D.
1,2	.9	3,6	.3
1,4	.1	4,6	.7
1,5	.4	4,7	.4
2,3	.9	5,6	.8
2,5	.6	5,7	.2
2,6	1.1	6,7	.9

2. (a) Derive the classical Economic Order Quantity (EOQ) formula:

$$\mathbf{y^* = [2KD/h]^{1/2}}$$

for the deterministic Inventory model where the demand rate D units/unit time is constant, an order of size y unit is received instantaneously, and the stock is depleted uniformly at a constant demand rate D. Here, K is the set up cost and h is the inventory holding cost per unit per unit time. You may make any other assumption as necessary.

(b) Suppose that, in the deterministic inventory model as in 2(a) above, there is a discount offer if the size of the order, y, exceeds a given limit q. More specifically, if the unit purchasing price, c, is given as

$$c = c_1 \text{ if } y \leq q$$
$$c_2 \text{ if } y > q \qquad c_1 > c_2$$

Explain mathematically, how, in this situation, you will proceed to optimize the order quantity, taking into consideration this price discount.

(c) The owner of a newsstand wants to determine the number of newspapers of TIMES OF INDIA stocked at the start of each day. The owner pays 2 rupees per copy and sell it at 4.5 rupees. Sale of the newspaper occurs between 6:00 and 7:00 A.M. Any copy left unsold at the end of the day can be recycled for an income of 0.30 rupee a copy. How many copies should the owner stock every morning, assuming the demand for the day can be described a normally distributed random variable with mean 250 copies and standard deviation 15 copies? [6+10+6]

3. (a) Consider the pure Birth model, where only arrivals occur (no departure) with inter arrival time exponential with mean inter arrival time $1/\lambda$. Prove that the number of arrivals, n, during a specified period t is a Poisson with mean λt. You may make *any assumptions that may be reasonable*.

(b) A fast food restaurant has one drive-in window. Cars arrive according to a Poisson distribution at the rate of 2 cars every 5 minutes. The space in front of the window can accommodate at most 10 cars, including the one being served. Others can wait outside this space if necessary. The service time per customer is exponential, with a mean of 1.5 minutes. Determine:

(i) The probability that the facility is idle.
(ii) The expected no. of customers waiting to be served.
(iii) The expected waiting time until a customer reaches the window to place order.
(iv) The probability that the waiting line will exceed the 10-space capacity.

[12+8]

4. An advertising agency is trying to determine a TV advertising schedule for Maruti Auto Company. Maruti has three goals:
    i. Its ads should be seen by at least 40 million high-income men (HIM).
    ii. Its ads should be seen by at least 60 million low-income people (LIP).
    iii. Its ads should be seen by at least 35 million high-income women (HIW).

   The advertising agency can purchase two types of ads: those shown during football games and those shown during soap operas. At most, $600,000 can be spent on ads. The advertising costs and potential audiences of a **one-minute ad of each type** are shown in the table. The agency must determine how many football ads and soap opera ads to purchase for Maruti.

Ad	Millions of Viewers			
	HIM	LIP	HIW	Cost (S)
Football	7	10	5	100,000
Soap opera	3	5	4	60,000

   Each million exposures by which Maruti falls short of the HIM goal costs Maruti a $200,000 penalty because of lost sales.

   Each million exposures by which Maruti falls short of the LIP goal costs Maruti a $100,000 penalty because of lost sales.

   Each million exposures by which Maruti falls short of the HIW goal costs Maruti a $500,000 penalty because of lost sales.

   Let x_1- number of minutes of ads shown during football games; x_2- number of minutes of ads shown during soap operas.

   Formulate a GLP that minimizes the cost incurred in deviating from Maruti's three goals.

5. A company manufactures two qualities of a product. The production of one unit of quality *A* product requires 100 man-hours and 16 units of raw material, whereas that of unit of quality *B* product requires 300 man-hours and 10 units of raw material. The profit per unit of quality *A* product and quality *B* product is Rs. 140 and Rs. 100, respectively. Further, on a daily basis, 12,000 man-hours of time and 800 units of raw material are available. The management feels that a daily profit margin of Rs. 7500 is a satisfactory rate of return, and even though production of quality *B* product is less profitable, it wishes to meet the present daily demand of 30 units of quality *B* product. If the target profit of Rs. 7500 is considered as a primary goal, and the demand of 30 units of quality *B* product is considered as a secondary goal, what production strategy should the company follow?
6. Patients arriving at a village dispensary are treated by a doctor on a first-come-first-serve basis. The inter-arrival time of the patients is known to be uniformly distributed between 0 and 80 minutes, while their service time is known to be uniformly distributed between 15 and 40 minutes. It is desired to simulate the system and determine the average time a patient has to be in the queue for getting service and the proportion of time the doctor would be idle. Carry out the simulation using the following sequences of random numbers. The numbers have been selected between 00 and 80 to estimate inter-arrival times between 15 and 40 to estimate the service times required by the patients.

   Series 1 : 07, 21, 12, 80, 08, 03, 32, 65, 43, 74
   Series 2: 23, 37, 16, 28, 30, 18, 25, 34, 19, 21.