Indian Statistical Institute, Bengaluru.

Operations Research-I

Time: 3 hour | *Mid-Term* | Maximum Marks: 30

1. (5 points) A farmer has recently acquired a 110 hectares piece of land. He has decided to grow Wheat and barley on that land. Due to the quality of the sun and the regions excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of \$10,000 and availability of 1,200 man-days during the planning horizon. Find the optimal solution and the optimal value using the graphical method.

2. (10 points) Find solution using Simplex method

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3$$

subject to

$$\begin{aligned} 2x_1 + 3x_2 &\leq 8 \\ 2x_2 + 5x_3 &\leq 10 \\ 3x_1 + 2x_2 + 4x_3 &\leq 15 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

3. (a) (7 points) A private company has manufacturing plants located in three cities A_1, A_2, A_3. Its market demand necessitates warehouses in four cities B, C, D, E. Let the maximum capacities of the plants and the demand requirements of the warehouses be as shown in Table below. In this table, the number in each cell represents the cost of transporting a unit from the particular plant to the given warehouse.

	B	C	D	E	Plant capacity
A_1	3	1	7	4	250
A_2	2	6	5	9	350
A_3	8	3	3	2	400
Warehouse demand	200	300	350	150	

Find the optimum solution to minimize the cost.

4. Consider the following problem:

$$Min\ c^t x$$

Question 4 continues on the next page. . .

Subject to

$$Ax \geq b$$
$$x \geq 0$$

where A is an $m \times n$ matrix, c and x are $n \times 1$ matrices, and b is an $m \times 1$ matrix.

(a) (1 point) Write the dual of the given problem.

(b) (1 point) State duality theorem.

(c) (1 point) State the complementary slackness theorem.

(d) (5 points) Using above parts, verify that (2,2,4,0) is an optimal solution to the following linear programming problem.
Maximize

$$2y_1 + 4y_2 + y_3 + y_4$$

Subject to

$$y_1 + 3y_2 + y_4 \leq 8$$
$$2y_1 + y_2 \leq 6$$
$$y_2 + y_3 + y_4 \leq 6$$
$$y_1 + y_2 + y_3 \leq 9$$
$$\text{and } y_j \geq 0 \ (1 \leq j \leq 4)$$